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PARAMETRIC STRESS ANALYSIS OF SPUR GEAR TOOTH

ASHISH KUMAR SINGH[1], MAHESH DEWANGAN[2]
[1*]P.G. Scholar [Design],Shri Shankaracharya Technical Campus
[2]Associate Professor, Deptt. of Mechanical Engineering, Shri Shankaracharya Technical Campus, Bhilai, India

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ASHISH KUMAR SINGH

ABSTRACT

This paper focuses on parametric stress analysis of a Spur gear has been done in order to optimize the performance parameters of spur gear during subjected to static and dynamic loads. Spur gear is most promising component in mechanical system for power transmission from one source to another source as per required one i.e. in higher or lower order. During this both process gear teeth are subjected to various load due to which stress are developed on the gear tooth. However, the Bending and Contact stress of the gear tooth are judged to be one of the chief contributors for the failure of gear in a gear system.

Thus, evaluation of stress parameters in gear design has become accepted area of research in gears to overcome from the failures and to optimal design of gears. In order to attain the goal a computational tool MATLAB is used to evaluate the bending stress, contact stress and contact ratio for a spur gear and several case studies are considered in which parameters that alters the stress (Bending and contact) are well investigated.

Moreover, the obtained results are compared with FEA result and literature and it shows good agreement. From this, several conclusions are drawn and discussed..

Key Words: Bending Stress, MATLAB, Contact stress, Pressure angle

INTRODUCTION

(In engineering and automobile, Gear tooth play very imperative role as an essential parts of engineering. The designers and manufacturers mostly adopt involute and evolute techniques to design a spur gear tooth. Due to higher degree of compactness and reliability Gears will overcome as a significant machine element designed for transmitting power in future technology.

Due to Advancement in science and technology, computers are well upgraded and becoming more influential device, that is why people tend to adopt numerical approach to develop theoretical model to envisage the effects. Due to less restrictive assumptions numerical methods are used since they provide more accurate solution. In order to attain accurate results correct model and solution method should be implemented along with reasonable computational time helps in validating result as per the trend.

Literature survey

There have been vast researches on gear scrutiny, and a large body of literature on gear modeling has been published. The gear stress analysis, transmission errors, and the prediction of gear dynamic loads, vibration, gear noise, and the optimal design for gear sets are always major concerns in gear design.

Until the mid 20th century all gear design was based upon Lewis original bending equation. Lewis based his analysis on a cantilever beam and assumed that failure will occur at the weakest point of this beam. Lewis considered the weakest point as the cross-section at the base of the spur gear.

Hertz calculated the contact pressure between two deformable cylinders.The contact pressure is mainly a function of the type of material in contact and the radius of curvature.

With this continuing trend of experimental bending stress analysis the American Gear Manufacturers Association (AGMA) published their own standard based on Lewis' original equation .Established in 1982 this standard is still widely used in gear design today. The bending stress is dependent on the geometry and shape of the gear tooth. [1]

Buckingham (1931) showed that two contacting parallel cylinders can be used to study contact stresses of spur gears with fair accuracy [2].

Ramamurti and Rao 1988 use fem and cyclic symmetry approach for the stress analysis of spur gear teeth. The contact line load at one such substructure leads to an asymmetric loading of the wheel as a whole. This force system is resolved into a finite Fourier series to calculate the static stresses. [3]

Vijayarangan and Ganesan 1993 uses 3 D FEA approach to obtain static stress analysis of composite gears and compared with mild steel gear and conclude that composite material is better for power transmission gears. [4]

Lu and Litvin [5] analyze the tooth surface contact and stresses for double circular-arc helical gear drives and FE method is use to investigate load share and contact ratio for aligned and misaligned gear.

Daniewicz and Moore 1998 increases fatigue life of gear by introducing compressive residual stresses is prestressing or presetting nad applied to AISI 1040 steel spur gear teeth were individually preset using a single tooth bending fatigue fixture. [6]

Woods and Daniewicz 1999 increases bending fatigue strength of carburized spur gear teeth using presetting and develop a model FEM in order to evaluate presetting on a gear tooth; his model is namely elastic–perfectly plastic.[7] The model has been verified experimentally and analytically. The place where Fatigue cracks originate this model helps to determine stress

Chien et. al 2002 Similar use roller test machine to study the spalling mechanism of spur gears for testing helical gearand explain explains how a subsurface crack is initiated and the influence of material properties on gear spalling life.[8]

Faydor et. al 2005 presents new computerized developments in design, generation, simulation of meshing, and stress analysis of gear drives and give numerical example for a developed theory.[9]

Yahaya and Ali [10, 11] designed S and C shaped transition curve and applied to design spur gear tooth. The design will be analyzed by using Finite Element Analysis (FEA). This analysis is used to find out the applicability of the tooth design and the gear material that chosen

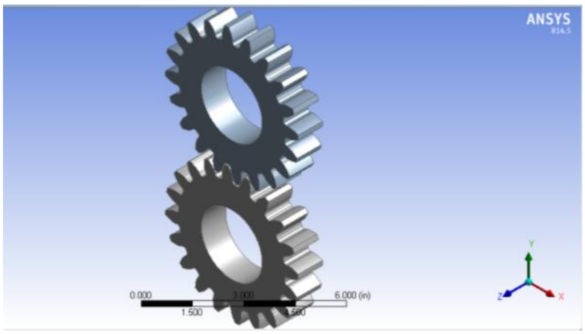
Lingamanaik and Chen 2012 uses metallurgical operation i.e Carburisation and quenching on automotive gears in order to improve wear properties by promoting martensite transformation and formation of a case hardened surface layer. [12]Since The martensite transformation causes a volumetric expansion which puts the surface into a 'compressive' residual stress state which promotes fatigue resistance.

Pandya and Parey 2013 uses the technique of conventional photo elasticity explore the possibility of using it as a supplementary technique to experimentally measure the variation of gear mesh stiffness and an innovative attempt has been made to calculate the variation of mesh stiffness for a pinion having a cracked tooth and a gear tooth with no crack of a spur gear pair. [13]

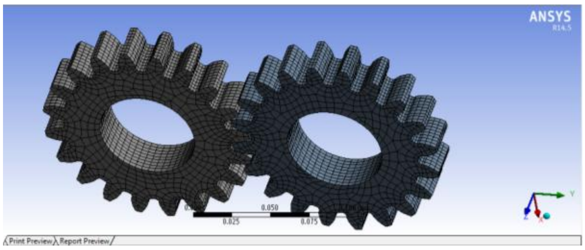
Sheng and Kahraman 2014 propose a physics model to calculate the micro-pitting behavior on contact surfaces of spur gears operating under the mixed lubrication condition. The transient mixed elasto hydrodynamic lubrication model of Li and Kahraman [14] forecast surface normal and tangential tractions, capturing the transient effects related with the time-varying contact radii, surface velocities and normal tooth force for spur gear.

Mathematical Model

The geometry of the problem herein investigated is depicted in Fig. 1.



Spur Gear



Mesh Model

Figure 1 Model Configuration

The FEM Formulation

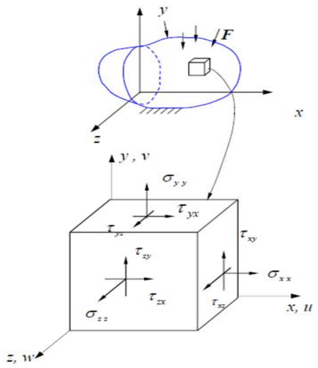


Figure 2 Infinitesimal element showing stress state [16].

Displacement

$$U=\{u(x,y,z),v(x,y,z),w(x,y,z)\}$$

Cauchy's Stress tensor =

$$\sigma=\begin{bmatrix}\sigma_{xx}\tau_{xy}\tau_{xz}\\ \tau_{yx}\sigma_{yy}\tau_{yz}\\ \tau_{zx}\tau_{zy}\sigma_{zz}\end{bmatrix}$$

The strain-stress relations (Hooke's law)for isotropic materials are given by:

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=\frac{1}{E}\begin{bmatrix}1 & -\nu & -\nu & 0 & 0 & 0\\ -\nu & 1 & -\nu & 0 & 0 & 0\\ -\nu & -\nu & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0\\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0\\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu)\end{bmatrix}\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{zx}\end{Bmatrix}$$

Strain-Displacement relations are:

$$\varepsilon_{xx}=\frac{\partial u}{\partial x},\varepsilon_{yy}=\frac{\partial v}{\partial y},\varepsilon_{zz}=\frac{\partial w}{\partial z},\gamma_{xy}=\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y},$$

$$\gamma_{yz}=\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z},\gamma_{xz}=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}$$

$$\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+\frac{\partial\tau_{xz}}{\partial z}+X=0$$

$$\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\tau_{yz}}{\partial z}+Y=0$$

$$\frac{\partial\tau_{xz}}{\partial x}+\frac{\partial\tau_{yz}}{\partial y}+\frac{\partial\sigma_{zz}}{\partial z}+Z=0$$

$$(\lambda+G)\frac{\partial e}{\partial x}+G\nabla^2u+X=0$$

$$e=\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}$$

$$b=\sqrt{\frac{4F\left\{\frac{\left[1-\mu_1^2\right]}{E_1}+\frac{\left[1-\mu_2^2\right]}{E_2}\right\}}{\pi l\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}}$$

Stresses internal to the cylinder are given by

$$\sigma_x=-P_{\max}\left\{\left[1+\left(\frac{y}{b}\right)^2\right]^{\frac{1}{2}}\left[2-\left(1+\left(\frac{y}{b}\right)^2\right)^{-1}\right]-2.\frac{y}{b}\right\}$$

$$\sigma_y=-P_{\max}\left[1+\left(\frac{y}{b}\right)^2\right]^{\frac{-1}{2}}$$

Von-mises stress is given by

$$\sigma_{von}=\frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$$

The equation below is the AGMA bending stress equation for S.I specification of gears.

$$\sigma_b=\frac{F_tk_vk_mk_o}{bmj}$$

Lewis,
$$\sigma = \frac{F_t}{bpy}$$

According to Shigley [17], the fundamental equation for pitting resistance (contact stress) is

$$S_c = C_p \sqrt{\frac{W_t k_a k_s k_m k_f}{k_v dFI}}$$

$$CR = \frac{\sqrt{(r_p+\phi)^2 - r_p^2\cos^2\phi}}{\pi m\cos\phi} + \frac{\sqrt{(r_g+\phi)^2 - r_g^2\cos^2\phi} - (r_p+r_g)\sin\phi}{\pi m\cos\phi}$$

$$C_p = \sqrt{\frac{1}{\pi\left(\frac{\left[1-\mu_1^2\right]}{E_1} + \frac{\left[1-\mu_2^2\right]}{E_2}\right)}}$$

In figure 1 the spur gear model configuration has been shown along with the mesh model. The gear model is discretisized in 39127 nodes and 7208 elements and the boundary conditions are applied and the von-mises (Bending) stress is evaluated.

RESULTS AND DISCUSSIONS

This chapter gives results on Stress analysis of Spur gear with using ANSYS and Analytical MATLAB calculation. The parametric study of effect of face width, Pressure Angle, varying load, no. of teeth on Spur gear is carried out.

The MATLAB results are validated with literature and by Analytical calculation for a few cases are also illustrated.

Table 1 Validation of Von-Mises Stresses for Spur gear Models

Load (MN)	Reference [10] (MPa)	Reference [16] (MPa)	Present MATLAB (MPa)
5	0.69	0.685	0.689
70	9.7	9.58	9.689
800	112	110	112.1
900	125	123	122.76
1000	139	137	137.89

Table 2: Validation of Von-Mises (Bending) Stresses for Spur gear Models

No of teeth(N)	MATLAB Stresses(MPA)	3D Stresses (ANSYS)(MPA)
22	130.1847	131.53
23	126.8841	127.04
25	122.2941	123.89
28	120.4364	122.94
30	119.0751	120.13
34	117.4243	118.57

For the number of teeth (Z) = 22

$$\sigma_b = \frac{F_t}{bmJ} k_v k_o k_m = \frac{3000}{43\times 4\times 0.36174} \times 1.2\times 1.25\times 1.8 = 130.1847 MPa$$

For number of teeth (Z) = 23

$$\sigma_b = \frac{F_t}{bmJ} k_v k_o k_m = \frac{3000}{43\times 4\times 0.37115} \times 1.2\times 1.25\times 1.8 = 126.8841 MPa$$

For number of teeth (Z) = 28

$$\sigma_b = \frac{F_t}{bmJ} k_v k_o k_m = \frac{3000}{43\times 4\times .39102} \times 1.22 \times 1.25 \times 1.8 = 120.436MPa$$

For number of teeth (Z) = 30

$$\sigma_b = \frac{F_t}{bmJ} k_v k_o k_m = \frac{3000}{43\times 4\times 0.39549} \times 1.22 \times 1.25 \times 1.8 = 119.075MPa$$

For number of teeth (Z) =34

$$\sigma_b = \frac{F_t}{bmJ} k_v k_o k_m = \frac{3000}{43\times 4\times 0.401056} \times 1.22 \times 1.25 \times 1.8 = 117.4242MPa$$

From Table 1, 2 and Figure 3 shows the stress distribution in spur gear and Shows the comparison of results for different 3-D models and the corresponding MATLAB stress values and Present FEM values. From this it can be revealed that on comparing Analytical result with computational result shows good agreement. And it can also be concluded that on increasing number of teeth of spur gear Von-Mises (Bending) Stresses decreases.

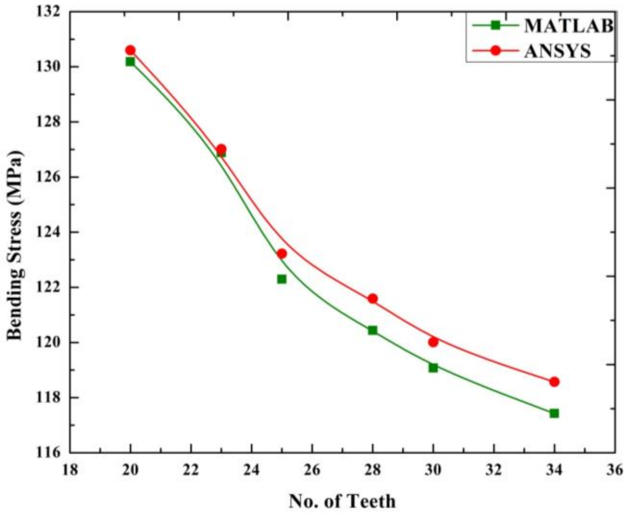


Figure 3 Variation of Stress with Number of Teeth of Spur Gear.

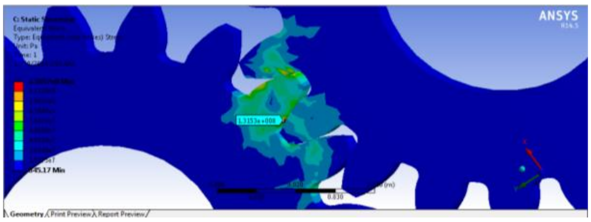


Figure 4 3-D Von-Mises Stress for Gear with 22 Teeth

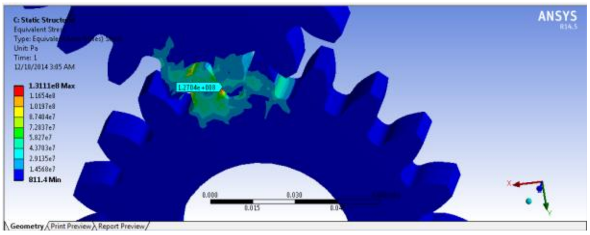


Figure 5 3-D Von-Mises Stress for Gear with 23 Teeth

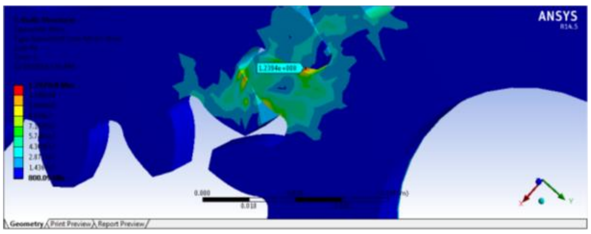


Figure6 3-D Von-Mises Stress for Gear with 25 Teeth

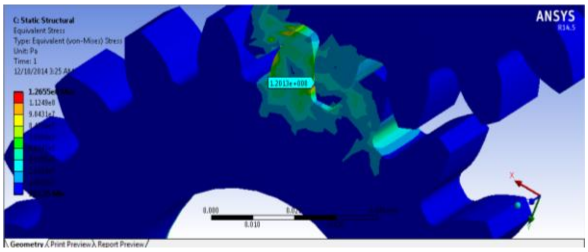


Figure 7 3-D Von-Mises Stress for Gear with 30 Teeth

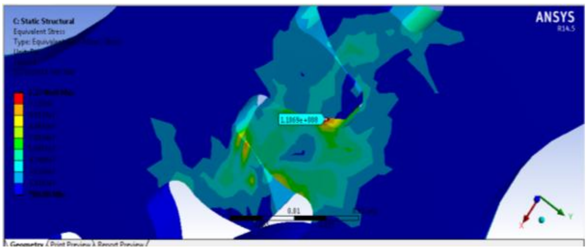


Figure 8 3-D Von-Mises Stress for Gear with 34 Teeth

From Figure 4-8 shows the stress distribution in spur gear 3-D models and the Table 6.1 Shows the comparison of results for different 3-D models and the corresponding AGMA stress values. From this it can be revealed that on comparing Analytical result with computational result shows good agreement. And it can also be concluded that on increasing number of teeth of spur gear Von-Mises (Bending) Stresses decreases.

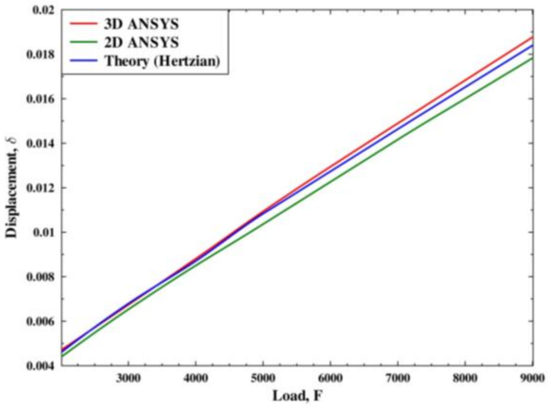


Figure 9 Variation of Displacement in varying load in spur gear.

Figure 9 shows the variation of displacement in varying load in spur gear in which a comparative result has been analyzed between 3D, 2D and Hertzian result. From this figure and result it can be conclude that on increasing force the displacement in gear goes on increasing linearly and there is good agreement between these results.

The 3D model showed good correlation with the more refined 2D model in displacement and contact surface pressure estimations, it failed in showing accurate stress distribution below the contacting surfaces. When analyzing parts that come into contact, the area just below the surface is the most critical area as far as the parts failure is concerned.

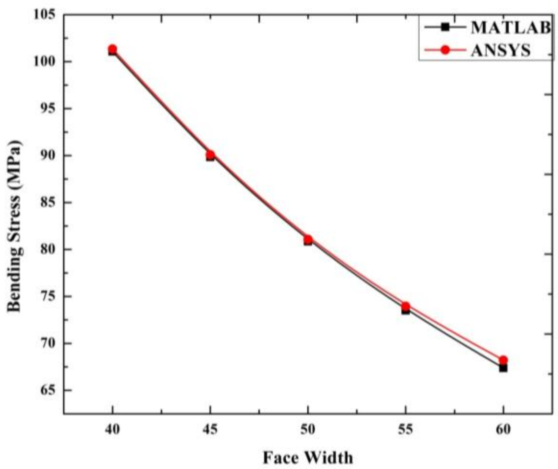


Figure 10 Variation of Bending Stress with respect to face width

Figure 10 shows the Variation of Bending Stress with respect to face width. It can seem the bending stress significantly decreases as the gear face width increase. The FEA Bending stress results shows a good agreement with the MATLAB result the variation of ±0.00263% is there.

It can also be conclude that on increasing face width, the bending stress spreads in more area and the load bearing capacity of the gear increases but on other hand the gear system become heavier.

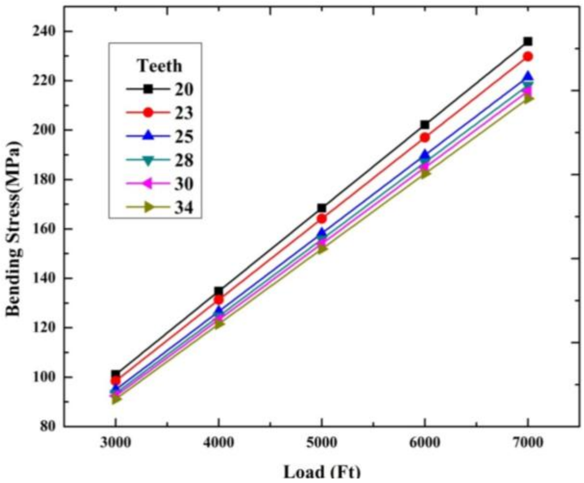


Figure 11 Variation of Bending Stress with respect to Tangential load and no. of Gear Teeth

In figure 11 shows the Variation of Bending Stress with respect to Tangential load and no. of Gear Teeth. It can be conclude that on increasing no. of teeth with respect to tangential load the bending stress increases linearly. As more the load more will be the bending. Such bending stress can be overcome by increasing no. of teeth.

Therefore during design of gear no. of teeth plays a crucial role in selection of gear performance parameters such dynamic factor kv, over load factor ko and j geometry factor.

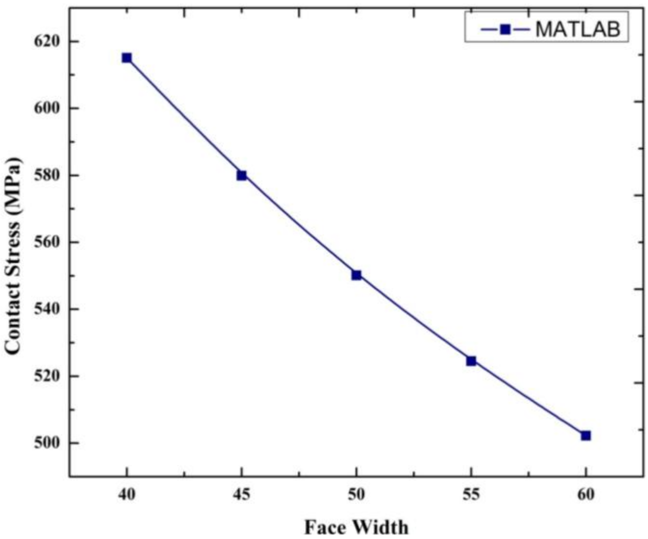


Figure 12 Variation of Contact Stress with respect to face width

Figure 12 shows the Variation of Contact Stress with respect to face width. it is clear from the fig. 6.10 that on increasing face width contact stress decreases.

It can also be revealed that around 18-20% contact stress decreases as face width33-35% increase.

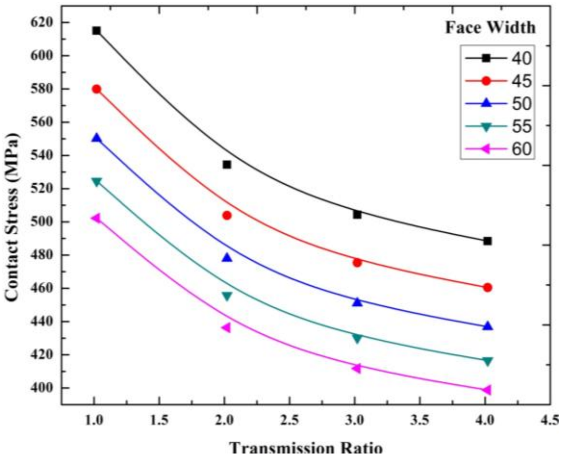


Figure 13 Variation of Contact Stress with respect to Transmission Ratio for different Face width

Figure 13 shows the Variation of Contact Stress with respect to Transmission Ratio and Face width. From this it can be concluded the on increasing no. of teeth in form of transmission ratio contact stress drastically decreases and the same treads is also noticed on increasing face width of gear.

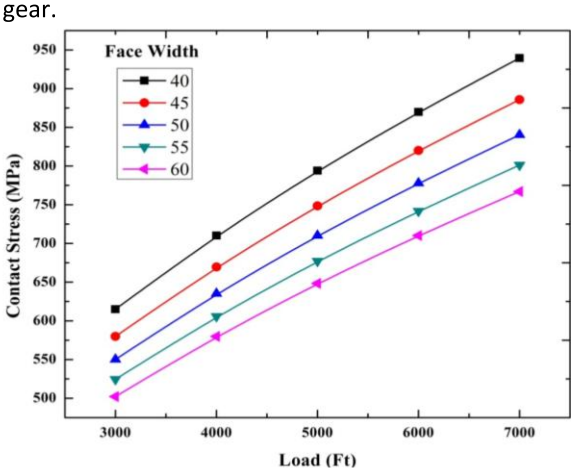


Figure 14 Variation of Contact Stress with respect to Tangential load for different Face width

Figure 14 shows the Variation of Contact Stress with respect to Tangential load and Face width, it seems that on increasing tangential load contact stress significantly increases but simultaneously at a same instance on increasing face width contact stress linearly decreases.

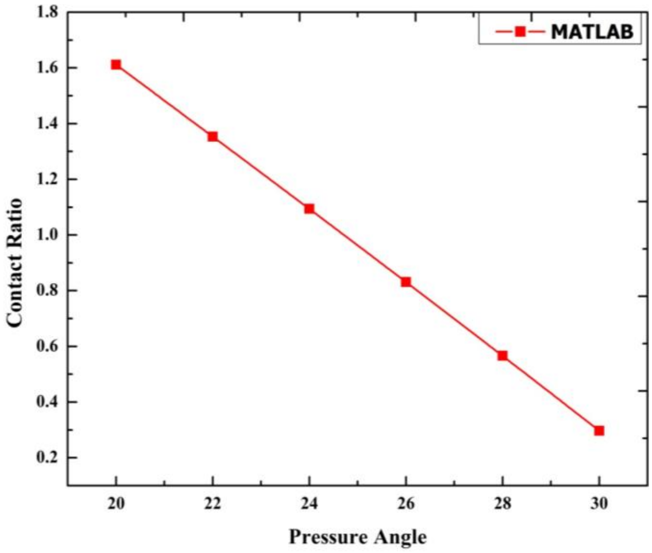


Figure 15 Variation of Contact ratio with respect to Pressure Angle

Figure 15 shows the Variation of Contact ratio with respect to Pressure Angle. From this it can be concluded that on increasing pressure angle contact ratio linearly decreases. The decline in contact ratio is remarkable as pressure angle increases i.e.16% decline from 20-22, 19% from 22-24 and 23.9% from 24-26 respectively

It can also be concluded that overlapping of tooth reduces and the loading capacity significantly increases.

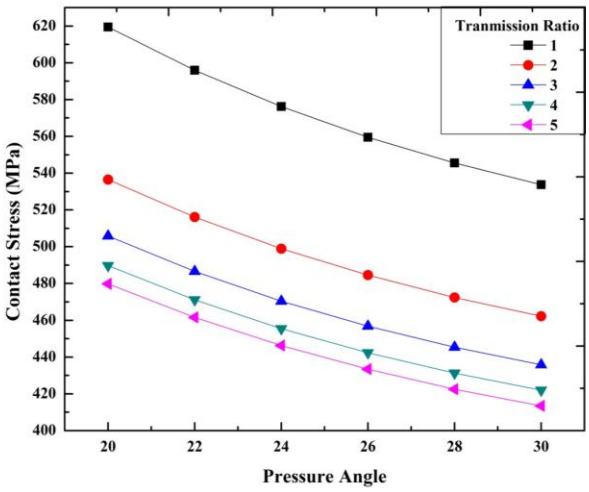


Figure16 Variation of Contact Stress with respect to Pressure Angle and Transmission Ratio

Figure16 show the Variation of Contact Stress with respect to Pressure Angle and Transmission Ratio. It can be concluded that on increasing pressure angle corresponding with transmission ratio the contact stress linearly decreases. Therefore, transmission ratio and pressure angle can be increases to the maximum limit as per the requirement in order to overcome from contact stress.

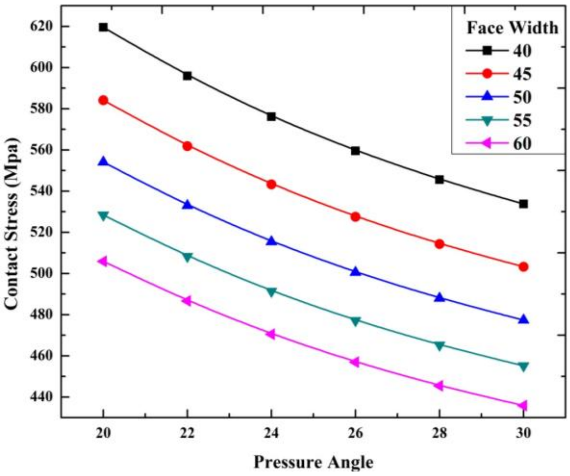


Figure 17 Variation of Contact Stress with respect to Pressure Angle and Face Width

Figure 17 shows the Variation of Contact Stress with respect to Pressure Angle and Face Width. From this it is seems that on increasing pressure angle along with Face width the contact stress significantly decreases.

It can also be revealed that pressure angle and face width are inversely proportional to contact stress. And the load carrying capacity becomes more.

CONCLUSION

It was observed that the stresses generated on spur gear teeth changes with the number of teeth.

A comparison of the results obtained from the FEM with those using the MATLAB (maximum bending stresses) and Hertz theory (maximum contact stresses) reveals thatthe maximum stresses predicted by the FEM are slightly higher than those predicted by the AGMA (MATLAB) and Hertz theory.

The variation between the MATLAB and ANSYS result is in the range of ±.0122 to ±.02014. Contact stress decreases linearly as pressure angle and face width increases correspondingly. Contact ratio and overlapping of gear get reduced as pressure angle increases. It is also found that on increasing transmission ratio corresponding pressure angle contact stress decreases. But this difference is of very small magnitude in comparison with the actual stress values and can be attributed to the difference in the theories involved. It can highly be recommended that in order to increase load carrying capacity of tooth pressure angle such be increased.

The height of the tooth is an important criterion and should be changed only if there are Space restrictions. A shorter gear tooth will produce more concentrated areas of stress which is ideally avoided, and should only be done if space is a major constraint.

Hardness of a tooth profile can improved can be prevented from pitting failure .i.e. a phenomenon in which a small particles are removed the surface of the tooth. This is due to high contact stress occurred between teeth during mating

The face width and transmission ratio are an important geometrical parameters during the design of gear. As it is expected, in this work the maximum bending stress decreases with increasing face width and it will be higher on gear of lower face width with higher transmission ratio. As a result, based on this finding if the material strength value is criterion then a gear with any desired transmission ratio with relatively larger face width is preferred.

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