International Journal of Engineering Research-Online
A Peer Reviewed International Journal
Articles available online http://www.ijoer.in

Vol.3., Issue.4., 2015
(July-Aug)

RESEARCH ARTICLE

ISSN: 2321-7758

PYTHAGOREAN TRIANGLE WITH HYPOTENUSE - $4\dfrac{Area}{Perimeter}$ AS A QUARTIC INTEGER

M.A.GOPALAN[1], K.GEETHA[2], MANJU SOMANATH[3]

[1]Professor, Department of Mathematics, Shrimathi Indira Gandhi College, Trichy,
[2]Assistant Professor, Department of Mathematics, Cauvery College for Women, Trichy,
[3]Assistant Professor, Department of Mathematics, National College, Trichy.

ABSTRACT

Infinitely many Pythagorean triangles such that each satisfying hypotenuse - $4\dfrac{Area}{Perimeter}$ is a quartic integer are obtained. A few interesting properties are also given.

Keywords: Pythagorean triangles, Quartic integer

Notations:

$t_{m,n}$ = Polygonal number of rank n with sides m

p_m^n = Pyramidal number of rank n with sides m

$ct_{m,n}$ = Centered Polygonal number of rank n with sides m

cp_m^n = Centered Pyramidal number of rank n with sides m

p_n = Pronic number

g_n = Gnomonic number

INTRODUCTION

The fascinating branch of mathematics is the theory of numbers where in Pythagorean triangles have been a matter of interest to various mathematicians and to the lovers of mathematics, because it is a treasure house in which the search for many hidden connection is a treasure hunt. For a rich variety of fascinating problems one may refer [1-9,12-18,20-23].A careful observer of patterns may note that there is a one to one correspondence between the polygonal numbers and the number of sides of the polygon.

In [10,11,19], special Pythagorean triangles connected with polygonal numbers and Nasty numbers are obtained. In [24] pairs of distinct Pythagorean triangles such that in each pair the difference between their perimeter is represented by i) $k\alpha^2$ ii) $k\alpha^n, n>2$ iii) $3p_{2N}^3$ iv) $12pt_{2N}$ are obtained. In [25], different methods of obtaining pairs of Pythagorean triangles which are such that, in each pair, the difference between the product of their generators is a perfect square.

In this communication, we present infinitely many Pythagorean triangles such that each satisfying hypotenuse - $4\dfrac{Area}{Perimeter}$ is a quartic integer are obtained. Also we have presented with suitable properties obtained from the generators of Pythagorean triangles.

Method of Analysis

It is well known that the Pythagorean triangle is represented by equation $x^2+y^2=z^2$ whose most cited solution is

$$x = 2pq,\ y = p^2 + q^2,\ z = p^2 - q^2,$$
$$p > q > 0 \qquad (1)$$

Here p, q are called generators of Pythagorean triangle.

The assumption

$$hypotenuse - 4\frac{Area}{Perimeter} = \alpha^4 \qquad (2)$$

gives

$$p^2 + 3q^2 - 2pq = \alpha^4 \qquad (3)$$

To start with it is noted that the triple of integers satisfy (3) is $\left(36R^2, 24R^2, 6R^2\right)$.

We solve (3) through different methods and obtain different choices for generators for p and q.

Substitute these values of p and q in equation (3), we obtain infinitely many Pythagorean triangles, each satisfying the relation (2).

Method 1:

Assume $p = u + nv$ and $q = nv$ (4)

in (3) gives, $u^2 + 2n^2v^2 = \alpha^4$ (5)

Let us assume $\alpha = (na)^2 + 2n^2b^2$ in (5) gives

$$u^2 + 2n^2v^2 = \left(\left((na)^2 + 2n^2b^2\right)^2\right)^2 \qquad (6)$$

On employing the method of factorization we get

$$\left(u + i\sqrt{2}nv\right)\left(u - i\sqrt{2}nv\right) = \left(\left(na + i\sqrt{2}nb\right)^2\left(na - i\sqrt{2}nb\right)^2\right)^2 \qquad (7)$$

On equating the positive and negative factors, we get

$$\left(u + i\sqrt{2}nv\right) = \left(\left(na + i\sqrt{2}nb\right)^2\right)^2$$

$$\left(u - i\sqrt{2}nv\right) = \left(\left(na - i\sqrt{2}nb\right)^2\right)^2$$

On equating real and imaginary parts, we have

$$u = n^4a^4 + 4n^4b^4 - 12a^2n^4b^2$$

$$nv = -8an^4b^3 + 4a^3n^4b$$

Substituting the values of u and nv in (4), the generators of p and q are given by

$$p = n^4a^4 + 4n^4b^4 - 12a^2n^4b^2 - 8an^4b^3 + 4a^3n^4b$$

$$q = -8an^4b^3 + 4a^3n^4b$$

For p and q to be generators of the Pythagorean triangles satisfy (2), the parameters a, b should satisfy the following conditions

1. $a^2 > 2b^2$
2. $\left(a^2 - 2b^2\right) > 8a^2b^2$

Numerical Example:

a	b	p	q	$Hypotenuse - 4\frac{Area}{Perimeter}$
4	1	292	224	$(18)^4$
5	1	789	460	$(27)^4$
7	2	2409	2296	$(57)^4$

A few interesting properties satisfied by generators are given below

1. $q(1,1,b) + b + 3cp_b^{16} = 0$
2. $p(n,2,1) + 12ncp_n^6 = 0$

Method 2:

Rewrite (6) as

$$u^2 + 2n^2v^2 = \left(\left((na)^2 + 2n^2b^2\right)^2\right)^2 * 1 \qquad (8)$$

Write 1 as

$$1 = \frac{\left(1 + i2\sqrt{2}\right)\left(1 - i2\sqrt{2}\right)}{9} \qquad (9)$$

Using (9) in (8) it is written in factorizable form as

$$\left(u + i\sqrt{2}nv\right)\left(u - i\sqrt{2}nv\right) = \left(\left(na + i\sqrt{2}nb\right)^2\left(na - i\sqrt{2}nb\right)^2\right)^2 \frac{\left(1 + i2\sqrt{2}\right)\left(1 - i2\sqrt{2}\right)}{9} \qquad (10)$$

On equating the positive and negative factors, we get

$$\left(u + i\sqrt{2}nv\right) = \left(\left(na + i\sqrt{2}nb\right)^2\right)^2 \frac{\left(1 + i2\sqrt{2}\right)}{3}$$

$$\left(u-i\sqrt{2}nv\right)=\left(\left(na-i\sqrt{2}nb\right)^2\right)^2\frac{\left(1-i2\sqrt{2}\right)}{3}$$

On equating real and imaginary parts, we obtain

$$\left.\begin{aligned} u&=\frac{1}{3}\left(n^4a^4+4n^4b^4-12n^4a^2b^2+32n^4ab^3-16n^4a^3b\right)\\ nv&=\frac{1}{3}\left(2n^4a^4+8n^4b^4-22n^4a^2b^2-8n^4ab^3+4n^4a^3b\right)\end{aligned}\right\} \quad (11)$$

Replacing a by 3A and b by 3B in the above equations (11), we get

$$u=27n^4A^4+108n^4B^4-324n^4A^2B^2+864n^4AB^3-432n^4A^3B$$

$$nv=54n^4A^4+216n^4B^4-648n^4A^2B^2-216n^4AB^3+108n^4A^3B$$

Substituting the values of u and nv in (4), the generators of p and q are given by

$$p=27\left(3n^4A^4+124n^4B^4-36n^4A^2B^2+24n^4AB^3-124n^4A^3B\right)$$

$$q=27\left(2n^4A^4+8n^4B^4-24n^4A^2B^2-8n^4AB^3+4n^4A^3B\right)$$

For p and q to be generators of the Pythagorean triangles satisfy (2), the parameters a, b should satisfy the following conditions

- $2A^3B+A^4+4B^4>4AB^3+12A^2B^2$
- $A^4+4B^4+32AB^3>12A^2B^2+16A^3B$

Numerical Example:

a	b	p	q	$Hypotenuse-4\frac{Area}{Perimeter}$
1	7	950373	413586	$(891)^4$
1	3	34101	6210	$(171)^4$
2	5	255636	21384	$(486)^4$

A few interesting properties satisfied by generators are given below

1. $p(n,1,2)-18225ncp_n^6=0$
2. $q(1,a,1)+27\left(t_{48,a}+14g_a+a+6-t_{6,a^2}-6cp_a^8\right)=0$

Remark:

It is worth to mention that instead of (9) one may have the following representations for 1 as

$$1=\frac{\left(7+i4\sqrt{2}\right)\left(7-i4\sqrt{2}\right)}{81}$$

$$=\frac{\left(7+i6\sqrt{2}\right)\left(7-i6\sqrt{2}\right)}{121}$$

$$=\frac{\left(17+i6\sqrt{2}\right)\left(17-i6\sqrt{2}\right)}{361}$$

Following the procedure presented in above methods, one may get the generators of p and q.

Method 3:

Write (5) as

$$2n^2v^2=\alpha^4-u^2$$

$$2vn*vn=\left(\alpha^2-u\right)\left(\alpha^2+u\right)$$

which is expressed in the form of ratio as

$$\frac{\alpha^2+u}{vn}=\frac{2vn}{\alpha^2-u}=\frac{A}{B},\ B\neq 0 \qquad (12)$$

This is equivalent to the following two equations

$$uB-Avn+B\alpha^2=0$$

$$uA+2Bvn-A\alpha^2=0$$

On solving the above equation by the method of cross multiplication we get,

$$\left.\begin{aligned} u&=A^2-2B^2\\ nv&=2AB\\ \alpha^2&=2B^2+A^2\end{aligned}\right\} \quad (13)$$

The above equation of the form

$$x^2=Dy^2+z^2$$

Therefore, $B=2rs$, $A=2r^2-s^2$ and

$$\alpha=2r^2+s^2$$

Substituting the values of A and B in (13), and the values of u and nv in (4), the generators of p and q are given by

$$p=4r^4+s^4-12r^2s^2+8r^3s-4rs^3$$

$$q=8r^3s-4rs^3$$

For p and q to be generators of the Pythagorean triangles satisfy (2), the parameters a, b should satisfy the following conditions

- $2r^2>s^2$
- $4r^4+s^4+4rs^3>12r^2s^2+4rs^3$

Numerical Example:

r	s	p	q	$Hypotenuse - 4\frac{Area}{Perimeter}$
3	1	421	204	$(19)^4$
5	2	3156	1840	$(54)^4$
6	3	5913	4536	$(81)^4$

A few interesting properties satisfied by generators are given below

1. $p(r,1)-q(r,1)+16r^2 = ct_{8,r}2$

2. $p(r,1)-t_{10,r}2 + t_{20,r} + 4g_r + 3 = q(r,1)$

Conclusion

One may search for Pythagorean triangles such that the hypotenuse - $4\frac{Area}{Perimeter}$ is a quartic integer. The generators is represented by special polygonal numbers and pyramidal numbers.

[1]. W.Sierpinski, Pythagorean triangles, Dover publications, INC, Newyork, 2003.

[2]. M.A.Gopalan, and S.Devibala, "On a Pythagorean problem", Acta Ciencia Indica, Vol. XXXII M, No 4, 1451-1452,2006.

[3]. M.A.Gopalan and A.Gnanam, "Pairs of Pythagorean triangles with equal perimeters", Impact J.Sci.Tech., Vol 1(2), 67-70, 2007.

[4]. M.A.Gopalan and S.Leelavathi, "Pythagorean triangle with 2 area/perimeter as a cubic integer", Bulletin of Pure and Applied Science, Vol.26E (No.2), 197-200,2007.

[5]. M.A.Gopalan and A.Gnanam, "A special Pythagorean problem", Acta Ciencia Indica, Vol. XXXIII M, No 4, 1435-1439,2007.

[6]. M.A.Gopalan, A.Gnanam and G.Janaki, "A Remarkable Pythagorean problem", Acta Ciencia Indica, Vol. XXXIII M, No 4, 1429-1434,2007.

[7]. M.A.Gopalan and G.Janaki, "Pythagorean triangle with area/perimeter as a special polygonal number", Bulletin of Pure and Applied Science, Vol.27E (No.2), 393-402, 2008.

[8]. M.A.Gopalan and S.Leelavathi, "Pythagorean triangle with area/perimeter as a square integer", International Journal of Mathematics, Computer sciences and information Technology, Vol.1, No.2, 199-204, 2008.

[9]. M.A.Gopalan and G.Janaki, "Pythagorean triangle with perimeter as Pentagonal number", Antarctica J.Math., Vol 5(2), 15-18, 2008.

[10]. M.A.Gopalan and G.Janaki, " Pythagorean triangle with nasty number as a leg", Journal of applied Mathematical Analysis and Applications, Vol 4,No 1-2,13- 17,2008.

[11]. M.A.Gopalan and S.Devibala, "Pythagorean triangle with Triangular number as a leg", Impact J.Sci.Tech., Vol 2(4), 195-199, 2008.

[12]. M.A.Gopalan and A.Vijayasankar, "Observations on a Pythagorean problem", Acta Ciencia Indica, Vol. XXXVI M, No 4, 517-520, 2010.

[13]. M.A.Gopalan and G.Sangeetha, "Pythagorean triangle with perimeter as triangular number",GJ-AMMS,Vol. 3, No 1-2,93-97,2010.

[14]. M.A.Gopalan and A.Gnanam, "Pythagorean triangles and Polygonal numbers", International Journal of Mathematical Sciences, Vol 9, No. 1-2, 211-215,2010.

[15]. M.A.Gopalan and B.Sivakami, "Pythagorean triangle with hypotenuse minus 2(area/ perimeter) as a square integer", Archimedes J.Math., Vol 2(2), 153-166, 2012.

[16]. M.A.Gopalan and B.Sivakami, "Special Pythagorean triangles generated through the integral solutions of the equation $y^2 = (k^2 + 2k)x^2 + 1$",Diophantus J.Math., Vol 2(1), 25-30, 2013.

[17]. M.A.Gopalan , Manjusomanath and K.Geetha," Pythagorean triangle with area/perimeter as a Special polygonal number", IOSR-JM, Vol. 7(3),52-62,2013.

[18]. M.A.Gopalan and V.Geetha," Pythagorean triangle with area/perimeter as a Special polygonal number", IRJES, Vol.2(7),28-34,2013.

[19]. M.A.Gopalan V.Sangeetha and Manjusomanath, “Pythagorean triangle and Polygonal number”,Cayley J.Math., Vol 2(2),151-156, 2013.

[20]. K.Meena, S.Vidhyalakshmi, B.Geetha, A.Vijayasankar and M.A.Gopalan,”Relations between special polygonal numbers generated through the solutions of Pythagorean equation”,IJISM, Vol. 2(2),257-258,2014.

[21]. M. A. Gopalan, K.Geetha and Manjusomanath, “On the rational Diophantine triples and Quadruples “, International journal of Scientific research publications, 4(9), Pg.1-6, sep 2014.

[22]. M. A. Gopalan, K.Geetha and Manjusomanath, “Special Dio- 3 tuples” , Bulletin of Society for Mathematical Services & Standards, Vol. 3 No. 2 (2014), pp. 41-45.

[23]. M.A. Gopalan, K.Geetha and Manjusomanath, “Construction of Diophantine triples for polygonal numbers($t_{26,n}$ to $t_{35,n}$) and centered polygonal numbers ($ct_{26,n}$ to $ct_{35,n}$)”, International journal of Modern Science and Engineering Technology, Vol.1, issue.8, 88-93, 2014.

[24]. M. A. Gopalan, S. Vidhyalakshmi, N. Thiruniraselvi, R. Presenna, “On Pairs of Pythagorean Triangles –I”, IOSR Journal of Mathematics, Vol.11, Issue 1, Ver. IV, 15 - 17, Jan- Feb 2015.

[25]. M. A. Gopalan, K. Geetha and Manjusomanath, “Pairs of Pythagorean triangles and Diophantine tuples”, Proceedings of National Conference on Recent Developments on Emerging fields in Pure and Applied Mathematics, 160 – 168, Mar 12th and Mar 13th 2015.