



PERFORMANCE ANALYSIS OF DIFFERENT DENOISING METHODS FOR ECG SIGNAL

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ABSTRACT

Electrocardiogram (ECG) has always been the most basic tool for diagnosis. Various kinds of noises can contaminate the ECG signals which lead to incorrect diagnosis. In this project, common and important denoising methods, the proposed method are presented and applied on ECG signal contaminated with noise. These are: discrete wavelet transforms, RLS adaptive filtering, Savitzky-Golay filtering, Empirical Mode Decomposition (EMD). The proposed method is combination of DWT and RLS adaptive filtering. Their denoising performances are implemented, compared and analyzed in a Matlab environment.

Keywords— ECG Signal, Denoising, RLS, Savitzky-Golay, Wavelet, EMD, SNR, PRD.

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I. INTRODUCTION

Electrocardiography (ECG) is the recording of the electrical activities of the heart and it uses the primary measure for identifying various heart diseases and heart abnormalities. It is very important for screening and diagnosis of many diseases. However, the presence of noises in ECG signals will affect the visual diagnosis and feature extraction. For its importance, an ECG signal should be presented as clean and clear as possible.

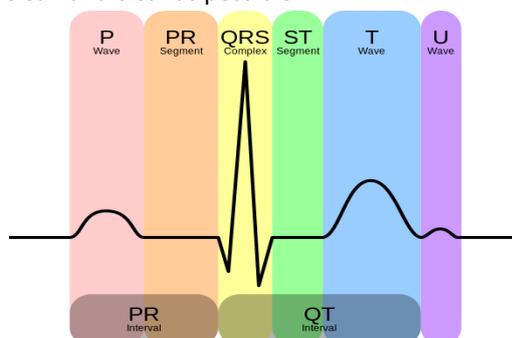


Fig.1 Ideal ECG signal.

As an electrical signal, ECG is susceptible to different kinds of noise. The main sources of this noise are electrical activities of other body muscles, baseline shift because of respiration, poor contact of electrodes, and equipment or electronic devices. There are different types of methods to denoise the ECG signals. As the ECG signal is a non stationary signal and the methods which are efficient are taken.

The wavelet transforms uses the filter bank concept and uses wavelet thresholding. Many digital filtering concepts were used for denoising. In FFT it loses its signal information in time domain and in IIR the drawbacks are it takes more memory, filtering time and also can't filter non-linear signals. But Adaptive filtering is used with ECG signals for noise cancellation. It gives faster filtering responses and gets less residual errors. In adaptive the LMS algorithm is not able to track the rapid changes in the ECG. The best adaptive filtering method is RLS adaptive filtering method for noise cancellation. Savitzky-Golay and EMD are used in smoothing ECG

Signals. In this paper, an ECG denoising method employing noise reduction algorithms of DWT and RLS adaptive filtering is presented. The simulation results show that the proposed method is able to reduce noise from the noisy ECG signals more accurately when compared to other methods.

II. RLS ADAPTIVE FILTERING

Adaptive filtering is one which can automatically design itself and can detect system variations in time using the input signal. It uses iterative computations to minimize the error between the desired and output signals. The basic ideal adaptive filter predicts the noise in the primary signal $x(n)$ and subtracts it. Here the primary signal is the combination of input signal and noise and the desired response is another ecg reference signal which is correlated to the primary signal.

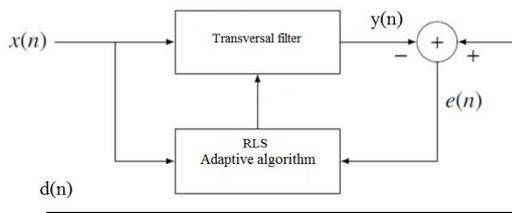


Fig.2 Adaptive filter.

$x(n)$ = input signal
 $y(n)$ = output signal
 $d(n)$ = desired signal
 $e(n)$ = error signal

$$y(n) = \sum_{i=0}^{L-1} w_i(n)x(n-i)$$

$$= W^T(n)X(n)$$

Where

$X(n) = [x(n), x(n-1), \dots, x(n-L+1)]$ is the filter tap input vector

$W(n) = [w_0(n), w_1(n), \dots, w_{L-1}(n)]^T$ is the filter tap weight vector

Here the objective of this filtering is to choose the weight vector of the filter so that the output signal is close to the estimate. Then

$$e(n) = d(n) - y(n)$$

$$e(n) = d(n) - W^T(n)X(n)$$

The modified or updated tap weight vector is

$$\hat{w}(n) = \hat{w}(n-1) + k(n)\xi^*(n)$$

Here $k(n)$ = kalman gain vector

$$\xi(n) = d(n) - \hat{w}^H(n-1)u(n)$$

And the gain vector is

$$K(n) = \frac{\Pi(n)}{\lambda + u^H(n)\pi(n)}$$

$$\pi(n) = P(n-1)u(n)$$

$$\hat{w}(n) = \hat{w}(n-1) + k(n)\xi^*(n)$$

$$P(n) = \lambda^{-1}P(n-1) - \lambda^{-1}k(n)u^H(n)P(n-1)$$

Where λ is forgetting factor ($0 < \lambda < 1$)

$P(n)$ is inverse correlation matrix of input signal

The initial value of $p(n)$ is

$$P(0) = \delta^{-1}I$$

I is identity matrix and δ is regularization factor.

III. DISCRETE WAVELET TRANSFORM

Wavelet transform is used to process the non stationary signals. It represents both the time and frequency representations. It allows only change in time extension not in shape of the signal. It represents the time functions in simple blocks called wavelets. A wavelet is an oscillation with amplitude that begins at zero, increases and then decreases back to zero. The wavelets are orthogonal, orthonormal, or biorthogonal, scalar or multi wavelets. In discrete wavelets for analyzing both the low and high frequency components, the filter bank tree decomposition and reconstruction is used. In this process the input signal is decomposed by repeatedly filtering through a pair of low pass filter (LPF) and a high pass filter (HPF). After filtering it was passed through down sampler at decomposition and up sampler at reconstruction. Sampling is done not to lose the information. This filtering is repeated base on the required levels. The discrete wavelet transform and its inverse is

$$f(t) = \sum_k c_j(k)\Phi_{j,k}(t) + \sum_j \sum_k d_j(k)\Psi_{j,k}(t)$$

Here $\varphi(t)$ is scaling function $\Psi(t)$ is wavelet function $h(n)$, $g(n)$ coefficients of LPF and HPF

$d_j(n)$, $c_L(n)$ are detail and approximation coefficients

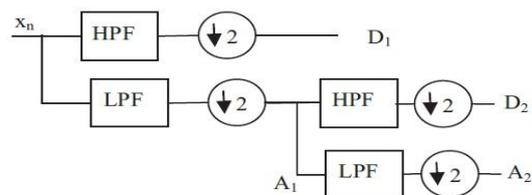


Fig.3 Decomposition.

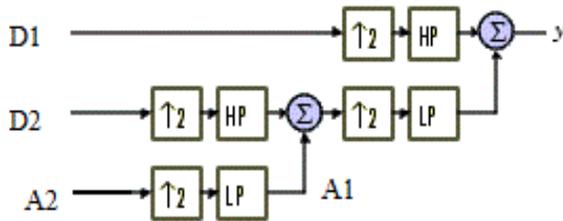


Fig.4 Reconstruction.

The signals of the LPF are the approximation coefficients (A_k) and the signals of the HPF are the detail coefficients (D_k). The approximation coefficients are related to the low frequency part of the signal, which contains the main features and information. And the detail coefficients are important to preserve the shape of the signal during reconstruction. The Donoho et.al algorithm is used for denoising by thresholding. Here the threshold value is determined by

$$T = \sigma\sqrt{2} \log(N)$$

N = number of wavelet coefficients.

σ = noise level estimation of 1st coefficient

There are two approaches in thresholding

Let d be the coefficient of noisy signal

$$\text{Hard } d = \begin{cases} d & \text{if } |d| > T \\ 0 & \text{else} \end{cases}$$

$$\text{Soft } d = \begin{cases} \text{sign}(d)(d - T) & \text{if } |d| > T \\ 0 & \text{else} \end{cases}$$

After thresholding the detail and approximation coefficients are reconstructed using IDWT.

IV. SAVITZKY GOLAY FILTERING

Savitzky golay filtering uses least square polynomial approximation for smoothening. Let there are $2M+1$ samples of a noisy signal $x[n]$. Then the coefficients of a polynomial is

$$p(n) = \sum_{k=0}^N a_k n^k$$

This requires mean square approximation error

$$\varepsilon_N = \sum_{n=-M}^M (p(n) - x[n])^2$$

Let a input signal vector has group of M samples at each side at the central point

$$x = [x_{-M}, x_{-M+1}, \dots, x_{-1}, x_0, x_1, \dots, x_M]^T$$

The smoothed output is obtained by combining the weighting coefficients or impulse response with the input samples and the interval $2M+1$ i.e. discrete convolution with input samples.

$$y(n) = \sum_{m=-M}^M h[m] x[n - m]$$

The vector of polynomial coefficients a can be computed as

$$a = [a_0, a_1, a_2, \dots, a_N]^T \quad a = (A^T A)^{-1} A^T X = Hx$$

where $A = \{n^i\}$, $i=0,1,\dots,N$

H =finite impulse response equivalent to least square polynomial approximation.

To calculate H ,

Set x to unit value centered in the interval $-M \leq n \leq M$

$$x = d = [0, \dots, 0, 1, 0, \dots, 0]^T$$

Thus we obtain the approximate polynomial coefficient vector

$$\tilde{a} = (A^T A)^{-1} A^T d$$

Where

$d = [0, 0, \dots, 0, 1, 0, \dots, 0]^T$ is a $(2M+1) \times 1$ column vector
 A^T is a $(N+1) \times (2M+1)$ matrix.

Now the 0^{th} row of the matrix $H = (A^T A)^{-1} A^T$ is

$$[h_{0,-M}, h_{0,-M+1}, \dots, h_{0,0}, \dots, h_{0,M-1}, h_{0,M}]$$

Which is equal to

$$[\tilde{p}(-M), \tilde{p}(-M+1), \dots, \tilde{p}(0), \dots, \tilde{p}(M)]$$

Where $\tilde{p}(n)$ is the polynomial that approximates d with least square error.

$$\tilde{p}(n) = \sum_{k=0}^N \tilde{a}_k n^k \quad -M \leq n \leq M.$$

Thus the impulse response of SG filter is

$$h[-n] = \tilde{p}(n).$$

V. EMPIRICAL MODE DECOMPOSITION (EMD)

EMD was introduced by huang et al for decomposing the given signal into a finite number of sub components. Using this any complicated data set can be decomposed into a finite and often small numbers of components. It is adaptive and even the basic functions are fully derived from the given data. The computation of EMD does not require any previously known value of the signal. It identifies the intrinsic oscillatory modes by their characters in time scale and according to it decomposes the signal into intrinsic mode functions (IMFs).

The functions are considered as IMFs they should satisfy two conditions:

- 1.) In the whole data set, the number of extrema and the number of zero-crossings must either be equal or differ by one.
- 2.) At any point, the mean value of the envelope defined by the local maxima and the local minima is zero.

The way of decomposing the input data signal into IMFs is known as "sifting" process.

The algorithm of shifting process is:

1. Calculate all the local maximas and minimas of the signal $x(n)$.

2. Find the mean (m_1) of maxima and minima of the data $x(n)$

$$m_1 = \frac{x_{max}(n) + x_{min}(n)}{2}$$

3. Subtract the mean from the data to get the first component $h_1(n)$

$$h_1(n) = x(n) - m_1$$

4. If $h_1(n)$ satisfies the conditions of IMF then

$h_1(n) = c_1(n)$ i.e. it is taken as the first IMF.

5. If $h_1(n)$ dissatisfies the conditions then 1 is considered as data for the next sifting process. And step 1 to 4 are repeated on $h_1(n)$

$$h_2(n) = h_1(n) - m_2$$

Here m_2 is mean of $h_1(n)$.

Then $h_2(n) = c_1(n)$

6. If $h_2(n)$ dissatisfies the conditions then standard difference (SD) is calculated from the consecutive components.

$$SD = \frac{\sum_{n=0}^N |h_{i-1}(n) - h_i(n)|^2}{h_{i-1}^2}$$

7. If SD is in the range of 0.2-0.3 then sifting process is terminated and then $h_1(n)$ is considered as $c_1(n)$

8. When $c_1(n)$ is obtained then it is subtracted from $x(n)$ to get the residue $r_1(n)$

$$r_1(n) = x(n) - c_1(n)$$

9. Now $r_1(n)$ is the new signal data for sifting process and from step 1 to 8 are done on $r_1(n)$ to get $r_2(n)$

$$r_2(n) = r_1(n) - c_2(n)$$

Therefore the residue signal obtained is

$$r_j(n) = r_{j-1}(n) - c_j(n)$$

If $r_j(n)$ becomes a constant function then the sifting process is terminated.

Then,

Original signal $x(n)$ is expressed as

$$x(n) = \sum_{i=1}^{L-1} c_i(n) + r_L(n)$$

Resulting residue or final residue is $r_L(n)$

Now the IMFs can be signal and noise or pure signal. So, let $\hat{c}_i(n)$ is a noiseless IMF, $c_i(n)$ is noisy IMF

and $n_i(n)$ is noise.

Then

$$c_i(n) = \hat{c}_i(n) + n_i(n)$$

$$\tilde{c}_i(n) = [(c_i(n), T)]$$

τ is the estimation function i.e. parameter T is applied on $c_i(n)$

$$T = \sigma\sqrt{2} \log(N)$$

N = number of wavelet coefficients.

σ = noise level estimation of 1st IMF.

Now thresholding is done by either hard or soft

$$\text{Hard } \tilde{c}_i(n) = \begin{cases} c_i(n) & \text{if } |c_i(n)| > T \\ 0 & \text{else} \end{cases}$$

Soft

$$\tilde{c}_i(n) = \begin{cases} \text{sign}(c_i(n))(|c_i(n)| - T) & \text{if } |c_i(n)| > T \\ 0 & \text{else} \end{cases}$$

Now the denoised signal is

$$\tilde{x}(n) = \sum_{i=1}^{L-1} \tilde{c}_i(n) + r_L(n).$$

VI. PROPOSED METHOD

In the proposed method we combine the concepts of the discrete wavelet transforms and the RLS adaptive noise cancellation. In this the signal is first denoised using discrete wavelet transform. Then the output or the denoised signal from the DWT is applied to the RLS adaptive noise cancellation algorithm. Here in DWT we used the 'db4' wavelet and for thresholding soft thresholding is used. Because the 'db4' and soft thresholding is the best pair for denoising a non stationary signal. The signal is first decomposed and then wavelet transform is applied and then thresholding was applied and then it was reconstructed by inverse wavelet transform.

wavelet transform

$$d_j(n) = (f, \Psi_{j,n}) = \frac{1}{\sqrt{M}} \sum_k g(2n-k) a_{j-1}(n)$$

$$c_j(n) = (f, \Phi_{j,n}) = \frac{1}{\sqrt{M}} \sum_k h(2n-k) c_{j-1}(n)$$

Inverse transform

$$f(t) = \sum_k c_j(k) \Phi_{j,k}(t) + \sum_j \sum_k d_j(k) \Psi_{j,k}(t)$$

Now it was applied to the RLS algorithm

$$k(n) = \frac{\Pi(n)}{\lambda + u^H(n)\pi(n)}$$

$$\pi(n) = P(n-1)u(n)$$

$$\xi(n) = d(n) - \hat{w}^H(n-1)u(n)$$

$$\hat{w}(n) = \hat{w}(n-1) + k(n)\xi^*(n)$$

$$P(n) = \lambda^{-1} P(n-1) - \lambda^{-1} k(n) u^H(n) P(n-1)$$

VII. RESULT AND SIMULATION

The performances of the methods are determined by comparing the denoised signal with the original signal. The performances are found by the signal to noise ratio (SNR), percentage root mean square difference (PRD), noise power.

$$PRD = 100 * \sqrt{\frac{\sum_{n=1}^N (x(n) - \hat{x}(n))^2}{\sum_{n=1}^N x^2(n)}}$$

$$SNR = 10 \log_{10} \frac{\sum_{n=1}^N |y(n) - x(n)|^2}{\sum_{n=1}^N |\hat{x}(n) - x(n)|^2}$$

$$Noise\ power = x^2(n) - \hat{x}^2(n)$$

Here $x[n]$ is the original ECG signal, $y[n]$ is the noisy signal, $\hat{x}[n]$ is the denoised ECG signal and N is number of ECG samples in the signal.

If the value of SNR is high then the noise in the signal is low. If the PRD and noise power values are high then the noise is more in the signal.

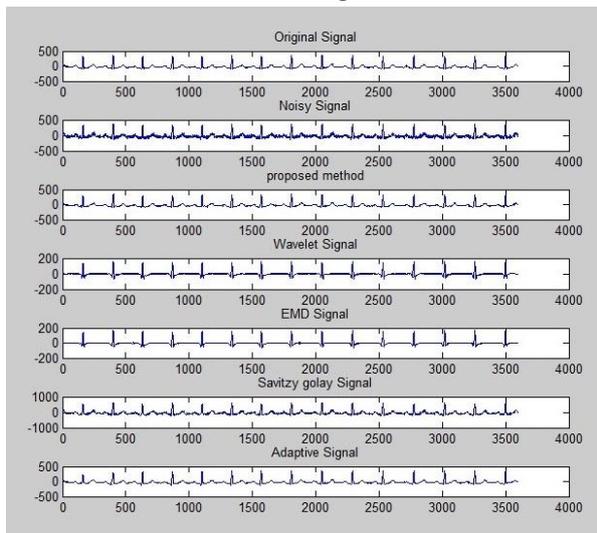


Fig.5 Denoised ECG signal from different methods.

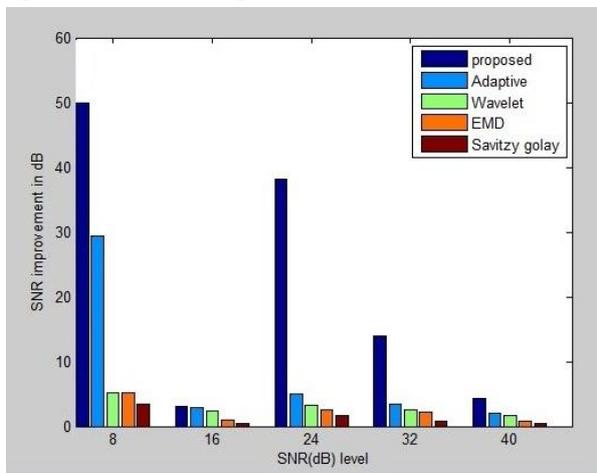


Fig.6 Comparison of SNR of different denoising methods.

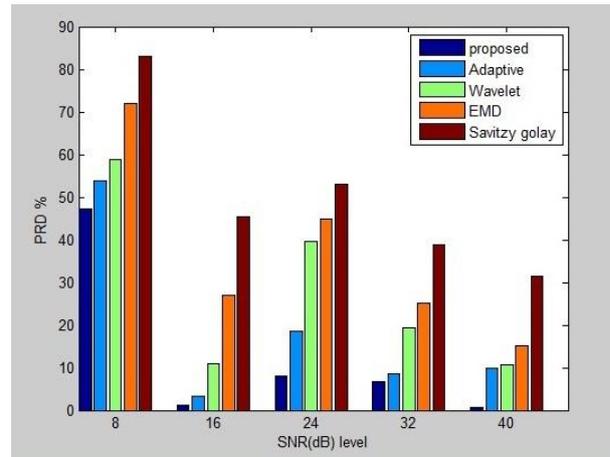


Fig.7 Comparison of PRD of different denoising methods

Here the Fig.5 shows the denoised ECGs obtained by different methods used and mentioned above. Fig. 6 shows the result of SNR comparison of the denoised signals. Fig.7 shows the PRD comparison. Fig.8 shows the noise power comparison.

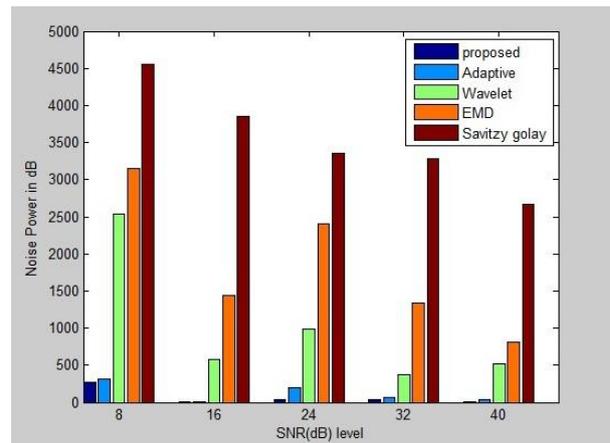


Fig.8 Comparison of Noise power of different denoising methods.

VIII. CONCLUSION

In this paper there are different method presented and applied on the ECG signal. The comparisons show that the proposed method has high SNR and less PRD and noise power. So, the proposed method reduces the noise efficiently when compared to others. As the wavelets perform well on non stationary ECG signal and then it is applied to RLS algorithm which converges faster and adjusts the filter parameters to environment to improve SNR and to reduce the mean between the original and denoised.

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