

RESEARCH ARTICLE

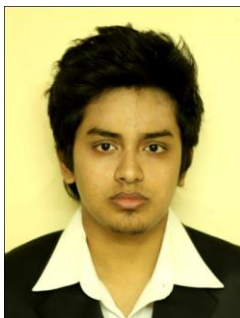


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## DESIGN PRINCIPLE OF SCHEFFLER SOLAR CONCENTRATOR FOR INDEPENDENT PARABOLIC CURVE LENGTH, APERTURE AREA AND INDEPENDENT RECEIVER POSITION

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### ABSTRACT

Scheffler solar concentrator is a device that concentrates thermal energy from sunlight to a fixed focus. It is the lateral section of a paraboloid. A receiver is placed in the focus. The device tracks the direct radiation of sunlight by necessary daily & seasonal rotation using either a manual or an automatic tracking system. In this paper design principle of the scheffler dis for an independent parabolic curve length, aperture area and independent focal distance from origin is described on an equinox day. The paper also describes the approach of changing the dish geometry if the receiver position or the fixed holding point of the device is needed to change. The design approach developed in this paper is validated comparing with the design of a standard  $8\text{m}^2$  (aperture area) scheffler reflectors.

**Keywords:** Scheffler concentrator, Solar energy, Design Principle, Parabolic reflector

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### INTRODUCTION

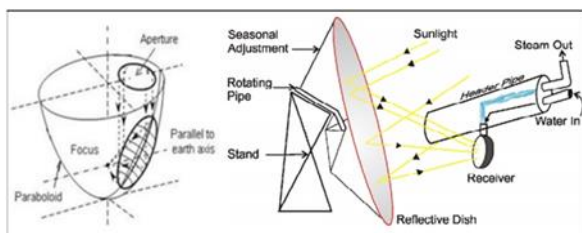
Solar energy has the height energy potential among the all renewable energy resources in the world. The Earth receives 174,000 terawatts (TW) of incoming solar radiation (insolation) at the upper atmosphere. Approximately 30% is reflected back to space while the rest is absorbed by clouds, oceans and land masses [Wikipedia]. So, practically there is immense of energy potential available from sun but we use very less of this input.

There are very few technology invented converting this solar energy to useful high grade and thermal energy. The available techniques are 1) Solar photovoltaic system 2) Solar concentrated power. The first one is widely used technique of solar energy that converts solar radiation into electricity. This system can work with both direct and disuse radiation of sun. The setup cost is still expensive and

maintenance is tough for an underdeveloped country.

The second technique is concentrated solar energy. This technique is getting popular day by day because of its low manufacturing cost and ease of maintenance. In this technique solar energy is received by a device that concentrates the energy to a receiver with maximum possible concentrating ratio. Numerous design of such kind of device has already been adopted in different part of the world. The devices are designed considering the application. High temperature collectors are used for industrial purpose or power generation purpose. The temperature is concentrated using different types of mirrors, highly polished metal sheets, and lenses. As the temperature increases, different forms of conversion become practical. Up to  $600\text{ }^\circ\text{C}$ , steam turbines, standard technology, have efficiency up to 41%. Above  $600\text{ }^\circ\text{C}$ , gas turbines can be more

efficient. Higher temperatures are problematic because different materials and techniques are needed. One proposal for very high temperatures is to use liquid fluoride salts operating between 700 °C to 800 °C, using multi-stage turbine systems to achieve 50% or more thermal efficiencies [1].



**Fig: 1 (working principle of Scheffler Solar Concentrator). [8]**

Devices for low and medium temperature are made concerning applications like water heating, drying, baking, cooking, distillation, and other non-industrial and domestic purpose. For medium temperature application scheffler reflector type concentrator is used in many different place of the world. The device is a section by an inclined plane from a 3D paraboloid. It is fixed to a point is called the fixed holding point of the device. The device is attached to this point and rotates bi-axially. So the joint on the fixed holding point should ensure these two types of rotation of the device. One is east to west or daily rotation and other is north to south or the seasonal rotation. Both manual and automatic tracking techniques are used to track the maximum intensity of the sunlight and to focus properly onto the receiver (Fig: 1). As the dish size increase its weight also increases. So it is advisable to setup the dish in such a way that ensures the nominal force required to rotate the device during both type of rotation. The size of scheffler dish is decided considering the application and power requirement. In the Gargi Hostel of MNIT Jaipur has a setup of steam generating system using scheffler dish. There are 6 dishes each of 16m<sup>2</sup> aperture area. The system cooks food for 500 students in the hostel. One essential part of scheffler system is the receiver. The main feature of the receiver is to absorb maximum possible amount of solar thermal energy from the reflector and transfer this heat to the working fluid with nominal loss of energy [2]. Receiver is generally a highly conductive metal like copper or aluminum.

One surface of the receiver is in contact with the working fluid other surface is coated with heat absorbing material usually black in color. In this paper a design procedure of scheffler dish is described for an independent curved length and independent focus. Design principle for other day can easily be identified using the design geometry of the equinox day.

## Material and Methods

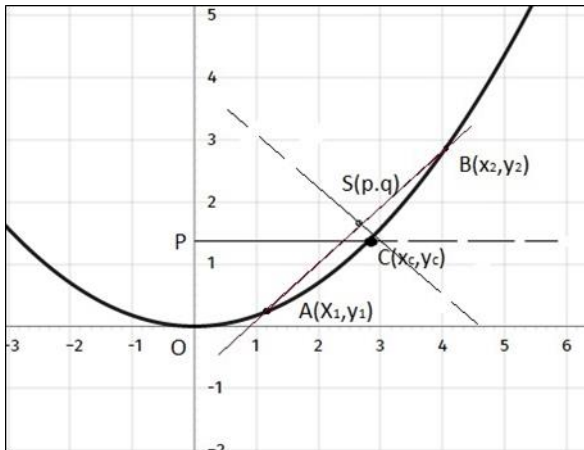
### Design Constraints:

Scheffler concentrator has a fixed focal point of concentrating the entire incident light on the concave parabolic surface of the concentrator. For necessary calculations of general design geometry of this device I have taken several constraints.

- The primary design calculation of the curved reflector frame is done for place in northern hemisphere on an equinox day. Where solar declination angle is zero.
- Equation of the parabolic curve is made considering the side view of the paraboloid. (As instructed the design of 8 m<sup>2</sup> scheffler concentrator [3])
- Curved length of the dish, fixed holding point and the focus point are assumed to be fixed for any other surrounding changes required i.e. change in daily and seasonal rotation of the dish, change in the shape of the dish etc. Line joining the focus point and fixed holding point of the dish is assumed parallel to the horizontal axis.
- Daily and seasonal rotation will rotate around the axis that is inclined to an angle equal to the latitude angle of the place where the dish will be kept. This assumption is made for the ease of tracking the solar radiation accurately.

### Design approach of the reflector geometry on an equinox day:

Consider the side view of the paraboloid. It will be a parabola with a fix focus P (Fig: 2). Lateral section of the paraboloid is the scheffler dish. For designing the dish parameter for equinox day I assumed earlier that the line joining the focus point and fixed holding point of the dish is parallel to the



**Fig: 2 (Design of parabolic structure of scheffler dish on equinox day)**

horizontal axis X. So on an equinox day, solar radiation will reflect by the fix point of the dish towards focus making a right angle with horizontal axis. Now, the equation of the parabola passing through the origin O (0, 0) and symmetric by Y axis is

$$x^2 = 4fy \quad (1)$$

Where, (f) is the focal distance (focus) from origin O.

Differentiating equation (1)

$$\frac{dy}{dx} = \frac{x}{2f}$$

At the fix point C(x<sub>c</sub>, y<sub>c</sub>),  $\frac{dy}{dx} = \tan 45 = 1$

So, at point C,  $1 = \frac{x_c}{2f}$  or,  $x_c = 2f$

Now,  $x^2 = 4fy$

$$x_c^2 = 4fy$$

$$4f^2 = 4fy_c$$

$$y_c = f$$

So, Co-ordinate of the fixed holding point is (x<sub>c</sub>, y<sub>c</sub>) = (2f, f)

The equation of the parabola can be written in another form i.e.

$$y = mx^2 + c$$

Where, (m) is the slope of the parabola at any point, (c) is the Y intercept. For equinox day the parabola is passing through the origin. So Y intercept will be zero (c=0). So the equation of the curve on equinox will be

$$y = mx^2 \quad (2)$$

Comparing equation (i) and (ii) we get,  $m = \frac{1}{4f}$

Now, equation of parabola

$$y = mx^2$$

$$\Rightarrow dy = 2mxdx$$

$\therefore dl = \sqrt{dx^2 + dy^2}$  [7] (This leads to calculate the curved length of the dish.)

$$= \sqrt{dx^2 + 4m^2x^2dx^2}$$

$$dl = \sqrt{1 + 4m^2x^2}dx \quad (3)$$

$\Rightarrow$  Curved length of the parabola

$$L = \int_{x_1}^{x_2} dl = \int_{x_1}^{x_2} \sqrt{1 + 4m^2x^2} dx \quad (4)$$

Where x<sub>1</sub>, x<sub>2</sub> are the x co-ordinate of two extreme points (A), (B) of the parabolic curve.

To move the reflector with a minimum force for any kind of seasonal and daily rotation the fixed holding point must be in at the center of mass of the dish. Means the fixed holding point and center of mass point of the dish will be same point. As the structure of the scheffler reflector is uniform so the center of mass and its center of gravity indicates the same point. So, point C (x<sub>c</sub>, y<sub>c</sub>) ( the fixed holding point) will be the center of gravity of the reflector.

Now, the x co-ordinate of the point C

$$x_c = \frac{1}{\int_{x_1}^{x_2} dL} \int_{x_1}^{x_2} x dL \quad [6]$$

$$x_c = \frac{1}{L} \int_{x_1}^{x_2} x \sqrt{1 + 4m^2x^2} dx$$

{From equation 3 & 4} (5)

Again, total curved length L= upper part length (from fixed holding point to upper extreme point) + lower part length (from fixed holding point to lower extreme point)

L =

$$\int_{x_c}^{x_2} \sqrt{1 + 4m^2x^2} dx +$$

$$\int_{x_1}^{x_c} \sqrt{1 + 4m^2x^2} dx = \int_{x_1}^{x_2} \sqrt{1 + 4m^2x^2} dx$$

Now,

$$\int \sqrt{1 + 4m^2x^2} dx = \int \sqrt{4m^2 \left( \frac{1}{4m^2} + x^2 \right)} dx$$

$$= 2m \int \sqrt{\left( \frac{1}{4m^2} + x^2 \right)} dx$$

$$= 2m \left[ \frac{x}{2} \sqrt{\frac{1}{4m^2} + x^2} + \frac{1}{8m^2} \ln \left( x + \sqrt{\frac{1}{4m^2} + x^2} \right) \right]$$

$$\text{So, } L = \int_{x_1}^{x_2} \sqrt{1 + 4m^2x^2} dx = 2m \left[ \frac{x}{2} \sqrt{\frac{1}{4m^2} + x^2} + \right.$$

$$\left. \frac{1}{8m^2} \ln \left( x + \sqrt{\frac{1}{4m^2} + x^2} \right) \right]_{x_1}^{x_2}$$

Thus getting,

$$\frac{L}{m} = x_2 \sqrt{\frac{1}{4m^2} + x_2^2} + \frac{1}{4m^2} \ln \left( x_2 + \sqrt{\frac{1}{4m^2} + x_2^2} \right) -$$

$$x_1 \sqrt{\frac{1}{4m^2} + x_1^2} - \frac{1}{4m^2} \ln \left( \sqrt{\frac{1}{4m^2} + x_1^2} + x_1 \right) \quad (6) \text{Again,}$$

from equation (5),  $x_c = 2f = \frac{1}{L} \int_{x_1}^{x_2} x \sqrt{1 + 4m^2 x^2} dx$

$$\text{Or, } 2fL = \int_{x_1}^{x_2} x \sqrt{1 + 4m^2 x^2} dx$$

$$2fL = \int_{x_1}^{x_2} x \sqrt{4m^2 \left( \frac{1}{4m^2} + x^2 \right)} dx$$

$$= 2m \int_{x_1}^{x_2} x \sqrt{\left( \frac{1}{4m^2} + x^2 \right)} dx$$

$$\text{Or, } \frac{fL}{m} = \int_{x_1}^{x_2} x \sqrt{\left( \frac{1}{4m^2} + x^2 \right)} dx$$

$$\text{Let, } \frac{1}{4m^2} + x^2 = z$$

$$\text{Or, } 2x dx = dz \text{ or, } x dx = \frac{dz}{2}$$

$$\begin{aligned} \text{Now, } \int x \sqrt{\frac{1}{4m^2} + x^2} dx &= \int \sqrt{z} \frac{dz}{2} = \frac{1}{2} \int \sqrt{z} dz = \frac{1}{3} z^{\frac{3}{2}} \\ &= \frac{1}{3} \left( \frac{1}{4m^2} + x^2 \right)^{\frac{3}{2}} \end{aligned}$$

(Replacing the value of z)

Thus find,

$$\frac{3fL}{m} = \left( \frac{1}{4m^2} + x_2^2 \right)^{\frac{3}{2}} - \left( \frac{1}{4m^2} + x_1^2 \right)^{\frac{3}{2}} \quad (7)$$

Solving equation (6) & (7) we can find the variable by

$x_1, x_2$

Now for equinox,  $x^2 = 4fy$

$$x_1^2 = 4fy_1 \text{ Or, } y_1 = \frac{x_1^2}{4f}$$

$$\text{Similarly, } y_2 = \frac{x_2^2}{4f}$$

∴ Two extreme points are  $(x_1, y_1); (x_2, y_2)$  are found for a curved length L.

We know that lateral section of the paraboloid will make an ellipse and horizontal projection (top view) of the ellipse will make a circle with radius equal to the radius of the minor axis of the ellipse (Fig: 3).

So, minor axis length of the dish on an equinox day,  $b = (x_2 - x_1)$

Major axis length,  $a$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = (x_2 -$$

$$x_1) \sqrt{1 + m^2(x_2 + x_1)^2} \quad (\text{Here, } y = mx^2)$$

Aperture area of the dish on same equinox day,  $A_{eq} =$

$$\frac{\pi ab}{4} = \pi \frac{(x_2 - x_1)^2 \sqrt{1 + m^2(x_2 + x_1)^2}}{4} \quad (8)$$

Co-ordinate of the midpoint of the elliptical rim,  $S =$

$$\left( \frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right) \equiv (p, q)$$

Central depth of the dish on equinox is denoted as SK. This central depth is nearly equal to the distance between the midpoint (S) and the fixed holding point (C), which is SC. So,  $(SK \approx SC)$ . The

surface of the dish can be compared to the surface of section of an ellipsoid with marginal value of error (Fig: 4) which major and minor axis is a, b and height is equal to  $(SK \approx SC)$ . Surface area of the dish is the required reflector area of the reflector. So, the reflector area is nearly equal to the surface area of that section of ellipsoid.

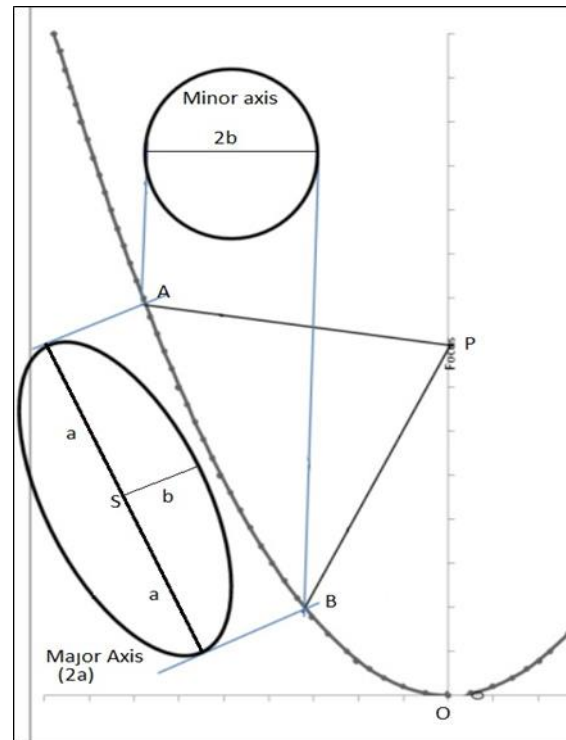


Fig: 3 (Major and minor axis)

So, Reflector area  $A_r \approx$  Surface area of the section of ellipsoid

$$\approx 2\pi \left\{ \frac{(ab)^{1.6} + (ac)^{1.6} + (bc)^{1.6}}{3} \right\}^{\frac{1}{1.6}} \quad (\text{KundThomsen Formula [5]})$$

Thus we can design a scheffler concentrator for a definite curved length and a self-defined focus. On the above discussion the design approach is only done for elliptical rim on an equinox day. Once we can decide the parameters on an equinox day we can set up an approach for the equation of the parabolic curve for all the days of year using the rotation matrix approach of the fixed point [3].

### Results and Discussion

The design procedure of elliptical frame of the scheffler dish on an equinox day; described above has two basic input or requirement. 1) Either curved length of the dish or the Aperture area. 2) Focal distance from origin.

Table 1: Inputs of Standard 8m<sup>2</sup> Scheffler dish [3]

Parameters	Value
f (focal distance from O)	1.43 m
x <sub>1</sub>	1.32
x <sub>2</sub>	4.06
y <sub>1</sub>	0.3046
y <sub>2</sub>	2.8817
x <sub>c</sub>	2.87
y <sub>c</sub>	1.43
m (slope at point P on equinox)	0.1748

For validating the equation I derived to design the reflector dish, I am referring the inputs same of an 8m<sup>2</sup> scheffler dish [3]. The input of this 8m<sup>2</sup> scheffler dish is listed in Table: 1. from the data of Table: 1, curved length of the dish is measured using equation (6). Thus getting a total curved length, L≈3.8030. Now, using this value of curved length Left side of equation (7) gives value ≈93.3. Right side of the equation (7) gives value≈ 91.23. The most possible reason for this little variation is underlying in the selection of end point coordinates of the 8m<sup>2</sup> scheffler dish. In the article the extreme points are chosen arbitrarily ([3] A. Munir et al. / Solar Energy 84 (2010) pg.1493). Any specific explanation is not given for what these coordinates had been chosen.

Taking the curved length L= 3.8030 and Focus=1.43 the corrected coordinate for this 8m<sup>2</sup> scheffler is identified using equation (6) & (7) is found 4.022 and 1.26. The solution is done by trial and error method but using computer program the values can be identified correctly. So the assumption of the coordinates of these extreme points of the referred article is quite accurate. Corresponding values of other parameters of the 8m<sup>2</sup> scheffler dish using equation (6) & (7) is listed in table no 2. However, the reflector surface area is not measured for 8m<sup>2</sup> dish [3]. This can be done using the Kund Thomsen Formula for calculating surface area, comparing the surface similar to an ellipsoid with nominal error. Finally using equation no (6),(7) & (8) we can figure out the design geometry of a scheffler dish on an equinox day for any kind of desired receiver position, aperture area or parabolic curved length.

Table 2: Modified parameters of 8m<sup>2</sup> Scheffler dish

Parameters	Value
f (focal distance from O)	1.43 m
x <sub>1</sub>	1.26
x <sub>2</sub>	4.022
y <sub>1</sub>	0.2775
y <sub>2</sub>	2.8280
x <sub>c</sub>	2.86
y <sub>c</sub>	1.43
m (slope on equinox)	0.1748
Upper curve length	1.8200
Lower curve length	1.9830
Total curve length	3.8030
Minor axis length	2.762 m
Major axis length	3.7594 m
p	2.641
q	1.427875
Central depth (SC ≈ SK)	0.219 m
A <sub>r</sub> (Reflector Area)	8.635 m <sup>2</sup>

### Conclusion

The design method of the scheffler dish for independent curved length and focus is described. In this paper this is done only for equinox day. Using this approach it is possible to construct any required size of scheffler dish. The approach is numerically proved with comparing a standard 8m<sup>2</sup> scheffler dishes. This approach also allows making necessary change in focus (place of the receiver) while not changing the whole scheffler dish but only changing its geometrical arrangement. Further work using this approach is possible to find out the curvature equation for all other days in year, to make any other curved concentrating device like ellipsoid concentrator, circular concentrator etc.

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### REFERENCES

- [1]. Charles W.Forseber, Per F. Petersen, H. Zhao, High-Temperature LiquidFluoride-Salt Closed-Brayton Cycle Solar Power Towers,

- Journal of Solar Energy Engineering, vol: 129, May 2007, Pg. (141-146).
- [2]. Kumar, N.S. Reddy K.S. Numerical investigation of natural convection heat loss in modified cavity receiver for fuzzy focal solar dish concentrator. Solar Energy 81 (2007) (846-855).
- [3]. A. Munir, O. Hensel, W. Scheffler. Design principle and calculations of a Scheffler fixed focus concentrator for medium temperature applications. Solar Energy 84 (2010) 1490–1502.
- [4]. Wolfgang Scheffler. INTRODUCTION TO THE REVOLUTIONARY DESIGN OF SCHEFFLER REFLECTORS. Available at [www.solare-bruecke.org](http://www.solare-bruecke.org).
- [5]. Kund Thomsen Formula. Available at <http://www.numericana.com/answer/ellipsoid.htm>.
- [6]. Center of Gravity of a Curve. Available at <http://www.maplesoft.com/applications/view.aspx?SID=32536&view=html>
- [7]. Arc Length of a Curve. Available at <http://www.maplesoft.com/applications/view.aspx?SID=32536&view=html>
- [8]. Scheffler Dish based Solar System Operations & Maintenance Manual. UNDP-GEF Project on Concentrated Solar Heat Ministry of New & Renewable Energy Government of India. November 2014.
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