

RESEARCH ARTICLE



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## LOGISTICS SERVICE PROVIDER SELECTION USING E-VIKOR METHOD IN INTUITIONISTIC FUZZY ENVIRONMENT

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### ABSTRACT

Decision making is the process of finding the best option among the feasible alternatives. In classical multiple criteria decision making methods, the ratings and the weights of the criteria are known precisely. However, if decision makers are not able to involve uncertainty in the defining of linguistic variables based on fuzzy sets, the intuitionistic fuzzy set theory can do this job very well. In this paper, VIKOR method is extended in intuitionistic fuzzy environment, aiming at solving multiple-criteria decision making problems in which the weights of criteria and ratings of alternatives are taken as triangular intuitionistic fuzzy set. For application and verification, this study presents a logistic service provider selection problem for garment manufacturing company to verify our proposed method.

**Key words:** hesitant intuitionistic fuzzy set; VIKOR method; Multiple criteria decision making (MCDM)

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### 1. INTRODUCTION

Decision making is universal in any human activity, either complex or simple. Most of the complex real life problems are with conflicting multi-criteria. A lot of work has been done on these complex structured multi-criteria problems and many methods are proposed to deal with them. MCDM methods are an extensively applied tool for determining the best solution among several alternatives with multiple criteria or attributes. The procedures for determining the best solution to a MCDM problem include computing the utilities of alternatives and ranking these utilities. The alternative solution with the largest utility is considered to be the optimal solution. Due to the complex structure of the problem and conflicting nature of the criteria, a compromise solution for a

problem can help the decision maker to reach a final decision. Recently Opricovic and Tzeng [15] has developed VIKOR method for multi-attribute optimization of complex systems. It determines the compromise ranking list, the compromise solution, and the weight stability intervals for preference stability of the compromise solution obtained with initial given weights. The method focuses on ranking and selecting from a set of alternatives in the presence of conflicting attributes. The VIKOR method provides a maximum group utility for the majority and a minimum of an individual regret for the opponent. It introduces the multi-attribute ranking indexes based on the particular measure of closeness to the ideal solution. Further in 2007, they[16] extended VIKOR method with stability analysis determining the weight stability intervals

and with trade-offs analysis. Tong et al. (2007)[18] have used VIKOR to optimize multi response process. As the complexity of the problem increases, impreciseness and vagueness in the data of the corresponding problem also increases.

Zadeh (1965)[23] proposed the idea of fuzzy sets to deal with these uncertainties. As Fuzzy set theory (Zimmerman, 1983, 1987) [25, 26] came into existence, many extensions of fuzzy sets also have appeared over the time and traditional fuzzy decision making models have been extended to include these extended fuzzy type descriptions. One among these extensions of fuzzy sets is Intuitionistic Fuzzy Sets (IFSs) (Atanassov, 1986) [3] playing an important role in decision making and have gained popularity in recent years. In IFS theory sum of degree of membership and degree of non-membership do not simply equals one as in the conventional fuzzy sets. Such an extended definition helps more adequately to represent situations when decision maker obtain from expressing their assessments. By this way, IFSs provide a richer tool to grasp imprecision than the conventional fuzzy sets. This feature of IFSs has led to extend VIKOR in intuitionistic fuzzy (IF)-environment.

In order to achieve the above purposes, this paper's organization structure is as follows. In the next section, we represent the IFS, HIFEs, IHFSs, the traditional VIKOR method. In section 3, we extend the traditional VIKOR method based on intuitionistic hesitant fuzzy set, and an approach is given. In Section 4, we give a numerical example to elaborate the effectiveness and feasibility of our approach.

## 2. Preliminaries

### Brief note on intuitionistic fuzzy sets

**Definition 2.1 [2].** Let  $X$  be a nonempty set. A fuzzy set  $A$  drawn from  $X$  is defined as

$A = \{x, \mu_A(x) / x \in X\}$  where the function  $\mu_A : X \rightarrow I$  is the membership function of the

**Definition 2.3 [7].** Let  $\tilde{n}_1 = \{\tilde{t}_1, \tilde{f}_1, \pi_1\}$  and  $\tilde{n}_2 = \{\tilde{t}_2, \tilde{f}_2, \pi_2\}$  be two HIFEs in a non-empty fixed set  $X$ , then

$$(1) \tilde{n}_1 \cup \tilde{n}_2 = \{\tilde{t}_1 \cup \tilde{t}_2, \tilde{f}_1 \cap \tilde{f}_2, \pi_1 \cap \pi_2\}, \quad (2)$$

$$(2) \tilde{n}_1 \cap \tilde{n}_2 = \{\tilde{t}_1 \cap \tilde{t}_2, \tilde{f}_1 \cup \tilde{f}_2, \pi_1 \cup \pi_2\}$$

Therefore, for two HIFEs  $\tilde{n}_1, \tilde{n}_2$  and a positive scale  $k > 0$ , the operations can be defined as follows:

fuzzy set  $A$ . Fuzzy set is a collection of objects with graded membership i.e., having degrees of membership.

**Definition 2.2 [2].** Let  $X$  be a nonempty set. An intuitionistic fuzzy set (IFS, in short)  $A$  in  $X$  is an object having the form

$$A = \left\{ x, \tilde{t}(x), \tilde{f}(x) / x \in X \right\} \text{ where the functions}$$

$\tilde{t} : X \rightarrow I$  and  $\tilde{f} : X \rightarrow I$  denote the *degree of*

*membership* (namely  $\tilde{t}(x)$ ) and the *degree of non-*

*membership* (namely  $\tilde{v}_A(x)$ ) of each element

$x \in X$  to the set  $A$  on a nonempty set  $X$  and

$$0 \leq \tilde{t}(x) + \tilde{f}(x) \leq 1 \text{ for each } x \in X.$$

Furthermore, we have

$$\pi(x) = 1 - \tilde{t}(x) - \tilde{f}(x)$$

called the **intuitionistic fuzzy set index or hesitation margin** of  $x$  in  $A$ .  $\pi(x)$  is the degree of

indeterminacy of  $x \in X$  to the IFS  $A$  and

$\pi(x) \in [0,1]$  i.e.,  $\pi(x) : X \rightarrow [0,1]$  and

$0 \leq \pi \leq 1 / x \in X$ .  $\pi(x)$  express the lack of

knowledge of whether  $x$  belongs to IFS  $A$  or not.

**Note:**

$$\tilde{t}(x) + \tilde{f}(x) + \pi(x) = 1$$

The  $\tilde{n} = \{\tilde{t}(x), \tilde{f}(x), \pi(x)\}$  is called *hesitant intuitionistic fuzzy element* (HIFE) which is the basic unit of the HIFS and is denoted by the symbol

$$\tilde{n} = \{\tilde{t}, \tilde{f}, \pi\}.$$

Then, some basic operations of HIFEs are defined as follows:

$$\begin{aligned}
 (1) \tilde{n}_1 \oplus \tilde{n}_2 &= \left\{ \tilde{t}_1 \oplus \tilde{t}_2, \tilde{f}_1 \otimes \tilde{f}_2, \pi_1 \otimes \pi_2 \right\} = \bigcup_{\substack{\tilde{\gamma}_i \in \tilde{t}_1, \tilde{\delta}_i \in f_i, \tilde{\eta}_i \in \pi_i, \\ \tilde{\gamma}_i \in \tilde{t}_2, \tilde{\delta}_i \in f_i, \tilde{\eta}_i \in \pi_i}} \{ \gamma_1 + \gamma_2 - \gamma_1 \gamma_2, \delta_1 \delta_2, \eta_1 \eta_2 \} \\
 (2) \tilde{n}_1 \otimes \tilde{n}_2 &= \left\{ \tilde{t}_1 \otimes \tilde{t}_2, \tilde{f}_1 \oplus \tilde{f}_2, \pi_1 \oplus \pi_2 \right\} = \bigcup_{\substack{\tilde{\gamma}_i \in \tilde{t}_1, \tilde{\delta}_i \in f_i, \tilde{\eta}_i \in \pi_i, \\ \tilde{\gamma}_i \in \tilde{t}_2, \tilde{\delta}_i \in f_i, \tilde{\eta}_i \in \pi_i}} \{ \gamma_1 \gamma_2, \delta_1 + \delta_2 - \delta_1 \delta_2, \eta_1 + \eta_2 - \eta_1 \eta_2 \} \\
 (3) k\tilde{n}_1 &= \bigcup_{\tilde{\gamma}_i \in \tilde{t}_1, \tilde{\delta}_i \in f_i, \tilde{\eta}_i \in \pi_i} \{ 1 - (1 - \gamma_1)^k, \delta_1^k, \eta_1^k \} \\
 (4) \tilde{n}_1^k &= \bigcup_{\tilde{\gamma}_i \in \tilde{t}_1, \tilde{\delta}_i \in f_i, \tilde{\eta}_i \in \pi_i} \{ \gamma_1^k, 1 - (1 - \delta_1)^k, 1 - (1 - \eta_1)^k \}
 \end{aligned}$$

**Proposition 2.4** [7]. Let  $\tilde{n}_1 = \{\tilde{t}_1, \tilde{f}_1, \pi_1\}$  and  $\tilde{n}_2 = \{\tilde{t}_2, \tilde{f}_2, \pi_2\}$  be two NHFEs in a non-empty fixed set  $X$ , and  $\eta, \eta_1, \eta_2 > 0$ , then we have

- (1)  $\tilde{n}_1 \oplus \tilde{n}_2 = \tilde{n}_2 \oplus \tilde{n}_1$ ;
- (2)  $\tilde{n}_1 \otimes \tilde{n}_2 = \tilde{n}_2 \otimes \tilde{n}_1$ ;
- (3)  $\eta(\tilde{n}_1 \oplus \tilde{n}_2) = \eta\tilde{n}_1 \oplus \eta\tilde{n}_2$ ;
- (4)  $\eta_1\tilde{n}_1 \oplus \eta_2\tilde{n}_2 = (\eta_1 + \eta_2)\tilde{n}_1$ ;
- (5)  $\tilde{n}_1^\eta \otimes \tilde{n}_2^\eta = (\tilde{n}_2 \otimes \tilde{n}_1)^\eta$ ;
- (6)  $\tilde{n}_1^{\eta_1} \otimes \tilde{n}_1^{\eta_2} = \tilde{n}_1^{\eta_1 + \eta_2}$ ;

**Definition 2.5**[7]. For an HIFE  $\tilde{n}$ ,

$$s(\tilde{n}) = \left[ \frac{1}{l} \sum_{i=1}^l \tilde{\gamma}_i + \frac{1}{p} \sum_{i=1}^p (1 - \tilde{\delta}_i) + \frac{1}{q} \sum_{i=1}^q (1 - \tilde{\eta}_i) \right] / 3$$

is called the score function of  $\tilde{n}$ , where  $l, p, q$  are the number of the values in  $\tilde{\gamma}, \tilde{\delta}, \tilde{\eta}$ , respectively. Obviously,  $s(\tilde{n})$  is a value belonged to  $[0,1]$ .

Suppose  $\tilde{n}_1 = \{\tilde{t}_1, \tilde{f}_1, \pi_1\}$  and  $\tilde{n}_2 = \{\tilde{t}_2, \tilde{f}_2, \pi_2\}$  are any two HIFEs, the comparison method of NHFEs is expressed as follows [17, 18]:

- (1) If  $S(\tilde{n}_1) > S(\tilde{n}_2)$ , then  $\tilde{n}_1 > \tilde{n}_2$ ;
- (2) If  $S(\tilde{n}_1) < S(\tilde{n}_2)$ , then  $\tilde{n}_1 < \tilde{n}_2$ ;
- (3) If  $S(\tilde{n}_1) = S(\tilde{n}_2)$ , then  $\tilde{n}_1 = \tilde{n}_2$ .

**Definition 2.6** [7]. Let  $\tilde{n}_1 = \{\tilde{t}_1, \tilde{f}_1, \pi_1\}$  and  $\tilde{n}_2 = \{\tilde{t}_2, \tilde{f}_2, \pi_2\}$  are any two HIFEs, then the normalized Hamming distance between  $\tilde{n}_1$  and  $\tilde{n}_2$  is defined as follows:

$$\begin{aligned}
 d(\tilde{n}_1, \tilde{n}_2) &= \|\tilde{n}_1 - \tilde{n}_2\| = \frac{1}{2} \left( |\tilde{\gamma}_1 - \tilde{\gamma}_2| + |\tilde{\delta}_1 - \tilde{\delta}_2| + |\tilde{\eta}_1 - \tilde{\eta}_2| \right) \\
 &= \frac{1}{2} \left( \left| \frac{1}{l} \sum_{j=1}^l \tilde{\gamma}_{1\sigma(j)} - \tilde{\gamma}_{2\sigma(j)} \right| + \left| \frac{1}{l} \sum_{j=1}^l \tilde{\delta}_{1\sigma(j)} - \tilde{\delta}_{2\sigma(j)} \right| + \left| \frac{1}{l} \sum_{j=1}^l \tilde{\eta}_{1\sigma(j)} - \tilde{\eta}_{2\sigma(j)} \right| \right)
 \end{aligned}$$

The hesitant fuzzy set (HFS)

**Definition 2.7** [15]. Let  $X$  be a non-empty fixed set, a HFS  $A$  on  $X$  is in terms of a function  $h_A(x)$  that when applied to  $X$  returns a subset of  $[0,1]$ , which can be denoted by the following mathematical symbol:

$$A = \left\{ \left\langle x, h_A(x) \right\rangle \mid x \in X \right\}$$

where  $h_A(x)$  is a set of some values in  $[0,1]$ , representing the possible membership degrees of the element  $x \in X$  to  $A$ . For convenience, we call  $h_A(x)$  a hesitant fuzzy element (HFE), denoted by  $h$ , which reads  $h = \{\gamma \mid \gamma \in h\}$ .

For any three HFEs  $h, h_1$  and  $h_2$ , Torra [21] defined some operations as follows:

- (1)  $h^c = \bigcup_{\gamma \in h} \{1 - \gamma\}$
- (2)  $h_1 \cup h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \max\{\gamma_1, \gamma_2\}$
- (3)  $h_1 \cap h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \min\{\gamma_1, \gamma_2\}$

After that, Xia and Xu [22] gave four operations about the HFEs  $h, h_1, h_2$  with a positive scale  $n$ :

- (1)  $h^n = \bigcup_{\gamma \in h} \{\gamma^n\}$
- (2)  $nh = \bigcup_{\gamma \in h} \{1 - (-\gamma)^n\}$
- (3)  $h_1 \oplus h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1 + \gamma_2 - \gamma_1\gamma_2\}$
- (4)  $h_1 \otimes h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1\gamma_2\}$

**Definition 2.8** [15]. Let  $h_1$  and  $h_2$  be two HFSs on  $X = \{x_1, x_2, \dots, x_n\}$ , then the hesitant normalized Hamming distance measure between  $h_1$  and  $h_2$  is defined as:

$$\|h_1 - h_2\| = \frac{1}{l} \sum_{j=1}^l |h_{1\sigma(j)} - h_{2\sigma(j)}|$$

where  $l(h)$  is the number of the elements in the  $h$ , in most cases,  $l(h_1) \neq l(h_2)$ , and for convenience, let  $l = \max\{l(h_1), l(h_2)\}$ . For operability, we should extend the shorter ones until both of them have the same length when compared. The best way to extend the shorter one is to add the same value in it. In fact, we can extend the shorter one by adding any value in it. The selection of the value mainly depends on the decision makers' risk preferences.

Optimists anticipate desirable outcomes and may add the maximum value, while pessimists expect unfavorable outcomes and may add the minimum value.

#### Logistics - Literature review

Logistics plays an important role in integrating the supply chain of industries. Because the market becomes more global, logistics is now seen as an important area where industries can decrease costs and improve their customer service quality [5].

Nowadays, many companies are searching to outsource their logistics operations to what they call as Third party Logistics Service Providers (LSPs) to introduce products and service innovations quickly to their markets. Therefore there is an increasing trend that manufacturing companies outsource their logistics activities to meet their increasing need for logistics services. This trend has increased importance of the concept of third party LSPs. In today's economic environment, many firms name third party LSPs as more qualified and economic in accomplishing their partial or all logistic requirements.

Outsourcing means that an organisation hires an outside organisation to provide a good or service that it traditionally had provided itself, because this third party is an "expert" in efficiently providing this good or service, while the organisation itself may not be [22]. Because of development of supply chain partnerships, cost reduction, restructuring of the company, success of the firms using contract logistics, globalisation, improvement of services, and efficient operations, companies need to outsource their logistic activities to 3PL service providers [6]. The outsourcing of logistic activities to third-party LSPs has now become a common practice. An LSP is defined as a provided or logistics services that performs the logistics functions on behalf of their clients. The LSP selection is a complex multicriteria decision making (MCDM) problem that includes both quantitative and qualitative criteria some of which can conflict each other and is vital in enhancing the competitiveness of companies. It is an important function of the logistics departments as it brings

significant savings for the organisation. While choosing the appropriate LSP, logistics managers

might be uncertain whether the selection will satisfy completely the needs of the organisation.

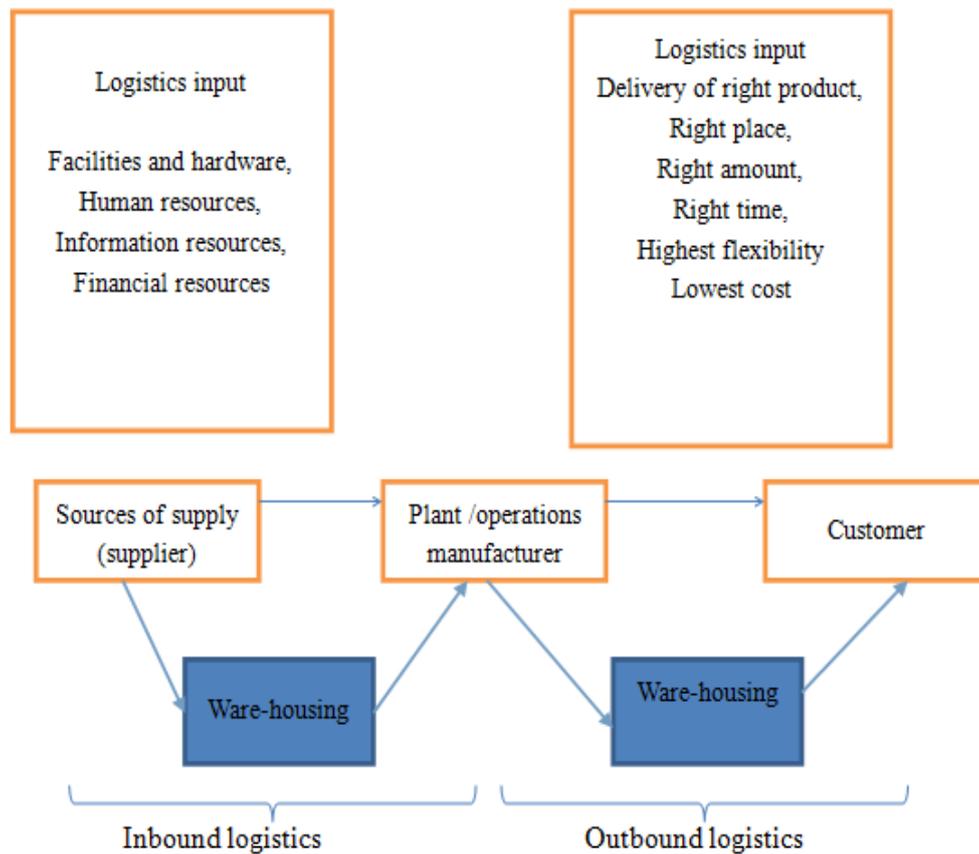


FIGURE 1: LOGISTICS MANAGEMENT

Because of some troubles in MCDM problems such as subjectivity, uncertainty, and ambiguity in assessment process, this study uses intuitionistic fuzzy analytic hierarchy process (FAHP) to establish the evaluation structure and calculate the importance weights of assessment criteria according to a group of decision-makers and applies intuitionistic fuzzy technique for order preference by similarity to an ideal solution (TOPSIS) to obtain the final ranking order of LSPs.

**Logistics Management.** According to definition by the Council of supply chain management professionals, it is accepted that logistics management is a part of supply chain management (SCM). It is the part "...that plans implements, and controls the efficient, effective forward and reverse flow and storage of goods, services, and related

information between the point of origin and the point of consumption in order to meet customers' requirements".

Logistics is an integration of information, transportation, material handling, stock and storage, and packaging operations. Logistics activities contain purchasing, transportation, quality, control, customs and insurance, handling, warehousing, inventory management, order processing, sales-demand forecast, logistics information management, distribution, labelling, packaging, fleet management, management of separate parts, product returns, and shipment planning. Logistics includes the flow of goods, services, and information related to movements of goods, services, and information related to movements of goods and services from the suppliers to a satisfied customer without waste.

Council of Logistics Management defined logistics as the process of planning, implementing, and controlling the efficient, cost effective flow and storage of raw materials, in-process inventory, finished goods, and related information from origin to consumption for the purpose of conforming to customer wants . According to this definition, logistics includes all activities related to the product, service, and information flow between supplier, manufacturer, and customer (Figure 1).

**Selecting Criteria for Evaluating Logistics Service Provider:** Deciding to use a third party LSP is a decision that depends on a variety of factors that differ from company to company. The decision to outsource certain business functions will depend on the company's plans, future objectives, product lines, expansion, acquisitions, and so forth .

Measures indicating the success of logistics management can be summarized as cost reduction, maximized on time delivery, minimized lead times, rapid respond to the market, higher flexibility, increased number of solution alternatives, improved information reliability, faster communication, minimized rate of consumption, damage and loss, minimized number of total inventory through the supply chain, transformation of fixed costs into variable costs, increased efficiency and productivity in logistics activities, reduction of logistics management expenses, focus on core competencies, improved customer relations, customer focus, and creating win-win relationships in the supply chain.

The needs of the firm can be satisfied by the third party logistics organization in optimum by defining the firm's goals and selection criteria. To know what metrics are used to evaluate the selection criteria of logistics service provider is an important issue. Generally, the companies have a variety of different characteristics related suppliers; but, if they use same methodology to evaluate the different types of suppliers, and the result cannot represent the real situation. Therefore, when determining the logistics service provider criteria, it should be considered that the criteria of selection differ in the different types of LSP .

*VIKOR method*

The VIKOR method was introduced for multi-criteria optimization problem. This method focuses on ranking and selecting from a set of alternatives, and determines compromise solution for a problem with conflicting criteria, which can help the decision makers to get a final solution. Here, the compromise solution is a feasible solution which is the closest to the ideal, and a compromise means an agreement established by mutual concessions [16]. It introduces the multi-criteria ranking index on the base of the particular measure of "closeness" to the "ideal" solution [14].The multi-criteria measure for compromise ranking is developed from the  $L_p$ -metric used as an aggregating function in a compromise programming method [15]. Development of the VIKOR method is started with the following form of  $L_p$ -metric:

$$L_{pi} = \left\{ \sum_{j=1}^n [(f_j^* - f_{ij}) / (f_j^* - f_j^-)]^p \right\}^{1/p} \quad 1 \leq p \leq \infty; i = 1, 2, 3, \dots, m.$$

In the VIKOR method  $L_{1,i}$  (as  $S_i$ ) and  $L_{\infty,i}$  (as  $R_i$ ) are used to formulate ranking measure. The solution obtained by  $\min S_i$  is with a maximum group utility ("majority" rule), and the solution obtained by  $\min R_i$  is with a minimum individual regret of the "opponent".

Assuming that each alternative is evaluated by each criterion function, the compromise ranking could be performed by comparing the measure of closeness to the ideal alternative. The various  $m$  alternatives are denoted as  $A_1, A_2, A_3, \dots, A_m$ . For alternative  $A_i$ , the rating of the  $j$  th aspect is denoted by  $f_{ij}$ , i.e.  $f_{ij}$  is the value of  $j$  th criterion function for the alternative  $A_i$ ;  $n$  is the number of criteria.

The compromise ranking algorithm of the VIKOR method has the following steps:

- (1) Determine the best  $f_j^*$  and the worst  $f_j^-$  values of all criterion functions  $j = 1, 2, \dots, n$ . If the  $j$ th function represents a benefit then:

$$f_j^* = \max_i f_{ij}, f_j^- = \min_i f_{ij}$$

(2) Compute the values  $S_i$  and  $R_i$ ;  $i=1,2,\dots,m$ , by these relations:

$$S_i = \sum_{j=1}^n w_j (f_j^* - f_{ij}) / (f_j^* - f_j^-),$$

$$R_i = \max_j w_j (f_j^* - f_{ij}) / (f_j^* - f_j^-),$$

where  $w_j$  are the weights of criteria, expressing their relative importance.

(3) Compute the values  $Q_i$ ;  $i=1,2,\dots,m$ , by the following relation:

$$Q_i = \nu(S_i - S^*) / (S^- - S^*) + (1 - \nu)(R_i - R^*) / (R^- - R^*)$$

where

$$S^* = \min_i S_i, S^- = \max_i S_i,$$

$$R^* = \min_i R_i, R^- = \max_i R_i$$

$\nu$  is introduced as weight of the strategy of "the majority of criteria" (or "the maximum group utility"), here suppose that  $\nu = 0.5$ .

(4) Rank the alternatives, sorting by the values  $S$ ,  $R$  and  $Q$  in decreasing order. The results are three ranking lists.

(5) Propose as a compromise solution the alternative  $A'$ , which is ranked the best by the measure  $Q$  (Minimum) if the following two conditions are satisfied:

C1. Acceptable advantage:  $Q(A'') - Q(A') \geq DQ$ , where  $A''$  is the alternative with second position in the ranking list by  $Q$ ;  $DQ = 1/(m-1)$ ;  $m$  is the number of alternatives.

C2. Acceptable stability in decision making: Alternative  $A'$  must also be the best ranked by  $S$  or/and  $R$ . This compromise solution is stable within a decision making process, which could be "voting by majority rule" (when  $\nu > 0.5$  is needed), or "by consensus"  $\nu \approx 0.5$ , or "with veto" ( $\nu < 0.5$ ). Here,  $\nu$  is the weight of the decision making strategy "the majority of criteria" (or "the maximum group utility").

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- Alternatives  $A'$  and  $A''$  are compromise solutions if only condition C2 is not satisfied, or

- Alternatives  $A', A'', \dots, A^{(M)}$  are compromise solutions if condition C1 is not satisfied;  $A^{(M)}$  is determined by the relation  $Q(A^{(M)}) - Q(A') < DQ$  for maximum  $M$  (the positions of these alternatives are "in closeness").

The best alternative, ranked by  $Q$ , is the one with the minimum value of  $Q$ . The main ranking result is the compromise ranking list of alternatives, and the compromise solution with the "advantage rate". VIKOR is an effective tool in multi-criteria decision making, particularly in a situation where the decision maker is not able, or does not know how to express his/her preference at the beginning of system design. The obtained compromise solution could be accepted by the decision makers because it provides a maximum "group utility" (represented by  $\min S$ ) of the "majority", and a minimum of the "individual regret" (represented by  $\min R$ ) of the "opponent". The compromise solutions could be the basis for negotiations, involving the decision maker's preference by criteria weights.

### 3. E-VIKOR method

According to these facts, when determining the exact values of the criteria is difficult or impossible, the hesitant intuitionistic fuzzy set is a very useful tool to deal with uncertainty in avoiding such issues in which each criteria can be described as a hesitant intuitionistic fuzzy set defined in terms of the opinions of decision makers and permits the membership having a set of possible values. Since, there is more appropriate to consider the values of the criteria as hesitant intuitionistic fuzzy element, where hesitant intuitionistic fuzzy elements are benefit criteria.

Therefore, in the present paper, we extend the VIKOR method to solve MADM problem with the hesitant intuitionistic fuzzy set information. To do this, suppose that a decision matrix, denoted by the intuitionistic hesitant fuzzy elements, has the following form (Table 1):

Table 1 Decision making matrix with the intuitionistic hesitant fuzzy set information

	$C_1$	$C_2$	...	$C_n$
--	-------	-------	-----	-------

$A_1$	$\tilde{n}_{11}$	$\tilde{n}_{12}$	...	$\tilde{n}_{1n}$
$A_2$	$\tilde{n}_{21}$	$\tilde{n}_{22}$	...	$\tilde{n}_{2n}$
...	...	...	...	...
$A_m$	$\tilde{n}_{m1}$	$\tilde{n}_{m2}$	...	$\tilde{n}_{mn}$

For a multiple criteria decision making problem, let  $A = \{A_1, A_2, \dots, A_m\}$  be a collection of  $m$  alternatives,  $C = \{C_1, C_2, \dots, C_m\}$  be a collection of  $n$  criteria, whose weight vector is  $w = (w_1, w_2, \dots, w_n)^T$

satisfying  $w_j \in [0,1], \sum_{j=1}^n w_j = 1$ . Suppose that

$\tilde{n}_{ij} = (\tilde{t}_{ij}, \tilde{l}_{ij}, \tilde{f}_{ij})$  is the evaluation value of the criteria  $C_j$  with respect to the alternative  $A_i$  which is expressed in the form of the hesitant intuitionistic fuzzy information, where  $\tilde{t}_{ij} = \{\tilde{\gamma}_{ij} | \tilde{\gamma}_{ij} \in \tilde{t}_{ij}\}$ ,  $\tilde{l}_{ij} = \{\tilde{\delta}_{ij} | \tilde{\delta}_{ij} \in \tilde{l}_{ij}\}$  and  $\tilde{f}_{ij} = \{\tilde{\eta}_{ij} | \tilde{\eta}_{ij} \in \tilde{f}_{ij}\}$  are three collections of some values in interval  $[0,1]$ , which represent the possible truth-membership hesitant degrees and falsity-membership hesitant degrees, and satisfies following limits:

$$\tilde{\gamma} \in [0,1], \eta \in [0,1], \text{ and}$$

$$0 \leq \sup \tilde{\gamma}^+ + \sup \tilde{\eta}^+ \leq 1, \text{ where}$$

$$\tilde{\gamma}^+ = \bigcup_{\tilde{\gamma}_i \in \tilde{t}_i} \max\{\gamma_{ij}\}, \tilde{\eta}^+ = \bigcup_{\tilde{\eta}_i \in \tilde{f}_i} \max\{\eta_{ij}\}.$$

Then we can rank the order of the alternatives.

The procedure of the proposed method as follows:

**Step 1.** Determine the positive ideal solution (PIS) and the negative ideal solution (NIS):

$$A^* = \{\tilde{n}_1^*, \dots, \tilde{n}_n^*\}, \text{ where}$$

$$\tilde{n}_j^* = \max\{S(\tilde{n}_{1j}), \dots, S(\tilde{n}_{mj})\}, j = 1, 2, \dots, n$$

$$A^- = \{\tilde{n}_1^-, \dots, \tilde{n}_n^-\}, \text{ where}$$

$$\tilde{n}_j^- = \min\{S(\tilde{n}_{1j}), \dots, S(\tilde{n}_{mj})\}, j = 1, 2, \dots, n$$

**Step 2.** In this step, compute  $S_i$  and  $R_i$  as below:

$$S_i = \sum_{j=1}^n w_j \frac{\|\tilde{n}_j^* - \tilde{n}_{ij}\|}{\|\tilde{n}_j^* - \tilde{n}_j^-\|}, i = 1, 2, \dots, m$$

$$R_i = \max_j w_j \frac{\|\tilde{n}_j^* - \tilde{n}_{ij}\|}{\|\tilde{n}_j^* - \tilde{n}_j^-\|}, i = 1, 2, \dots, m$$

**Step 3.** Compute the values  $Q_i : i = 1, 2, \dots, m$ , by the following relation:

$$Q_i = v(S_i - S^*) / (S^- - S^*) + (1-v)(R_i - R^*) / (R^- - R^*)$$

where

$$S^* = \min_i S_i, S^- = \max_i S_i,$$

$$R^* = \min_i R_i, R^- = \max_i R_i$$

where  $v$  is introduced as weight of the strategy of "the majority of criteria" (or "the maximum group utility"), here suppose that  $v = 0.5$

**Step 4.** Rank the alternatives, sorting by the values  $S, R$  and  $Q$  in decreasing order. The results are three ranking lists.

**Step 5.** Propose as a compromise solution the alternative  $A'$ , which is ranked the best by the measure  $Q$  (Minimum) if the following two conditions are satisfied:

$$C1. \text{ Acceptable advantage: } Q(A'') - Q(A') \geq DQ$$

, where  $A''$  is the alternative with second position in the ranking list by  $Q$ ;  $DQ = 1/(m-1)$ ;  $m$  is the number of alternatives.

**C2. Acceptable stability:** Alternative  $A'$  must also be the best ranked by  $S$  or/and  $R$ . This compromise solution is stable within a decision making process, which could be "voting by majority rule" (when  $v > 0.5$  is needed), or "by consensus"  $v \approx 0.5$ , or "with veto" ( $v < 0.5$ ). Here,  $v$  is the weight of the decision making strategy "the majority of criteria" (or "the maximum group utility").

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- (i) Alternatives  $A'$  and  $A''$  if only condition C2 is not satisfied, or
- (ii) Alternatives  $A', A'', \dots, A^{(M)}$  if condition C1 is not satisfied;  $A^{(M)}$  is determined by the relation  $Q(A^{(M)}) - Q(A') < DQ$  for maximum  $M$  (the positions of these alternatives are "in closeness").

#### 4. Case study

We consider an example to select logistic service provider for a garment manufacturing company. Four people working in the logistics department of the company were determined to select evaluation criteria, who are marked by

$A_i (i=1,2,3,4)$ , and they are measured by three criteria: (1)  $C_1$  ( On time delivery); (2)  $C_2$  (Price); (3)  $C_3$  (Firm reputation), and the evaluation values are denoted by IHFNs and their weight is

$w = (0.35, 0.25, 0.4)^T$ . The decision matrix  $R$  is shown in the Table 2.

Table 2 The hesitant intuitionistic fuzzy decision matrix

	$C_1$	$C_2$	$C_3$
$A_1$	{{0.4,0.4,0.5}, {0.4}}	{{0.5,0.6},{0.3,0.4}}	{{0.3},{0.5,0.6}}
$A_2$	{{0.7},{0.2,0.3}}	{{0.7},{0.3}}	{{0.7},{0.2}}
$A_3$	{{0.4,0.6},{0.3}}	{{0.6},{0.5}}	{{0.6},{0.3}}
$A_4$	{{0.8},{0.2}}	{{0.7},{0.2}}	{{0.5},{0.2,0.3}}

**Step 1.** Determine the positive ideal solution (PIS) and the negative ideal solution (NIS):

$$A^* = \{\tilde{n}_1^*, \tilde{n}_2^*, \tilde{n}_3^*\} = \{\{0.8\}, \{0.1\}, \{0.2\}\},$$

$$\{\{0.7\}, \{0.1\}, \{0.2\}\}, \{\{0.7\}, \{0.2\}, \{0.2\}\}$$

$$A^- = \{\tilde{n}_1^-, \tilde{n}_2^-, \tilde{n}_3^-\} = \{\{0.4, 0.6\}, \{0.4\}, \{0.3\}\},$$

$$\{\{0.6\}, \{0.3\}, \{0.5\}\}, \{\{0.3\}, \{0.2\}, \{0.5, 0.6\}\}$$

**Step 2.** In this step, compute  $S_i$  and  $R_i$  as below:

$$S_1 = \frac{w_1 \|n_1^* - n_{11}\|}{\|n_1^* - n_1^-\|} + \frac{w_2 \|n_2^* - n_{12}\|}{\|n_2^* - n_2^-\|} + \frac{w_3 \|n_3^* - n_{13}\|}{\|n_3^* - n_3^-\|} = 0.942$$

$$S_2 = \frac{w_1 \|n_1^* - n_{21}\|}{\|n_1^* - n_1^-\|} + \frac{w_2 \|n_2^* - n_{22}\|}{\|n_2^* - n_2^-\|} + \frac{w_3 \|n_3^* - n_{23}\|}{\|n_3^* - n_3^-\|} = 0.166$$

$$S_3 = \frac{w_1 \|n_1^* - n_{31}\|}{\|n_1^* - n_1^-\|} + \frac{w_2 \|n_2^* - n_{32}\|}{\|n_2^* - n_2^-\|} + \frac{w_3 \|n_3^* - n_{33}\|}{\|n_3^* - n_3^-\|} = 0.733$$

$$S_4 = \frac{w_1 \|n_1^* - n_{41}\|}{\|n_1^* - n_1^-\|} + \frac{w_2 \|n_2^* - n_{42}\|}{\|n_2^* - n_2^-\|} + \frac{w_3 \|n_3^* - n_{43}\|}{\|n_3^* - n_3^-\|} = 0.187$$

$$R_1 = \max_3 \left\{ \frac{w_1 \|n_1^* - n_{11}\|}{\|n_1^* - n_1^-\|}, \frac{w_2 \|n_2^* - n_{12}\|}{\|n_2^* - n_2^-\|}, \frac{w_3 \|n_3^* - n_{13}\|}{\|n_3^* - n_3^-\|} \right\} = 0.4$$

$$R_2 = \max_3 \left\{ \frac{w_1 \|n_1^* - n_{21}\|}{\|n_1^* - n_1^-\|}, \frac{w_2 \|n_2^* - n_{22}\|}{\|n_2^* - n_2^-\|}, \frac{w_3 \|n_3^* - n_{23}\|}{\|n_3^* - n_3^-\|} \right\} = 0.125$$

$$R_3 = \max_3 \left\{ \frac{w_1 \|n_1^* - n_{31}\|}{\|n_1^* - n_1^-\|}, \frac{w_2 \|n_2^* - n_{32}\|}{\|n_2^* - n_2^-\|}, \frac{w_3 \|n_3^* - n_{33}\|}{\|n_3^* - n_3^-\|} \right\} = 0.35$$

$$R_4 = \max_3 \left\{ \frac{w_1 \|n_1^* - n_{41}\|}{\|n_1^* - n_1^-\|}, \frac{w_2 \|n_2^* - n_{42}\|}{\|n_2^* - n_2^-\|}, \frac{w_3 \|n_3^* - n_{43}\|}{\|n_3^* - n_3^-\|} \right\} = 0.187$$

**Step 3.** Let  $v = 0.5$ , compute the values  $Q_i (i=1,2,3,4)$ :

$$Q_1 = 1, Q_2 = 0, Q_3 = 0.774, Q_4 = 0.126$$

**Step 4.** Rank the alternatives, sorting by the values  $S, R$  and  $Q$  in decreasing order. The results are three ranking lists, which is depicted in Table 3.

Table 3 The ranking and the compromise solutions.

	$A_1$	$A_2$	$A_3$	$A_4$	Ranking	Compromise solutions
$S$	0.942	0.166	0.733	0.187	$A_2 \succ A_4 \succ A_3 \succ A_1$	$A_2$
$R$	0.4	0.125	0.35	0.187	$A_2 \succ A_4 \succ A_3 \succ A_1$	$A_2$
$Q(v = 0.5)$	0.997	0.003	0.774	0.126	$A_2 \succ A_4 \succ A_3 \succ A_1$	$A_2$

**Step 5.** The ranking of alternatives by  $Q$  in decreasing order, the alternative with first position is  $A_2$  with  $Q(A_2) = 0.003$ , and  $A_4$  is the alternative with second position with  $Q(A_4) = 0.126$ . As  $DQ = 1/(m-1) = 1/(4-1) = 0.333$ , so  $Q(A_4) - Q(A_2) = 0.123 < 0.333$

Which is not satisfied  $Q(A_4) - Q(A_2) \geq \frac{1}{4-1}$ , but alternative  $A_2$  is the best ranked by  $S$  and  $R$ , which satisfies the condition two. By computing, we get:  
 $Q(A_4) - Q(A_2) = 0.123 < 0.333$   
 $Q(A_3) - Q(A_2) = 0.771 > 0.333$

so  $A_2$ ,  $A_4$  both compromise solution.

### 5. Conclusion

Hesitant intuitionistic fuzzy set is the generalization of intuitionistic set and the hesitant fuzzy set. Some operational laws, comparison rules of hesitant intuitionistic fuzzy set and the Hamming distance between two hesitant intuitionistic fuzzy numbers are defined. For multiple criteria decision making with hesitant intuitionistic fuzzy sets, the traditional VIKOR method is extended, and an approach is given. In this method, which is based on the particular measure of "closeness" to the "ideal" solution, using linear programming method during the process of decision-making, and order the hesitant fuzzy numbers by index of attitude and choose the alternatives under the acceptable advantage and the stability of the decision-making process to get a compromise solution, which achieving the maximum "group utility" and minimum of an "individual regret". This method has its own advantages compared with other multiple criteria decision making method based on distance, but it can only solve the decision making problems with criteria is hesitant intuitionistic fuzzy numbers and fixed weights, in the case of uncertain weights is universal in real life, which needs further study.

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