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PARAMETRIC INSTABILITY OF SIGMOID TIMOSHENKO BEAM ON VARIABLE ELASTIC FOUNDATION

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ABSTRACT

A sigmoid Timoshenko beam resting on parabolic elastic foundation is investigated for its parametric instability. Finite element method is used for simply supported end condition of the beam. The static part of the governing differential equation of the beam is solved to determine the shape functions to obtain accurate results. The effect of geometry, power index, foundation stiffness and foundation parameter on dynamic stability of the beam is investigated.

Key words:sigmoid, foundation parameter, foundation modulus, power index.

I. INTRODUCTION

Functionally graded material (FGM) consists of two or more dissimilar materials in which the volume fraction of constituent materials is varied continuously as a function of position along certain dimension(s) of the structure thereby varying its properties accordingly. FGMs can be used in several engineering sectors such as the aerospace, aircraft, automobile, defense industries, electronic and the biomedical sectors. Many machine and structural components in aforesaid sections can be modeled as beams[11]. A literature survey is carried out and an overview of the survey is given below.

Kocatu, T investigated the dynamic deflections of eccentrically prestressed viscoelastic Timoshenko beam subjected to a moving harmonic load with a constant velocity and also compared the obtained natural frequencies of the Timoshenko beams with previously published results based on the Timoshenko beam theory and exact results based on the Euler–Bernoulli beam theory[1]. Yunmin, Chen Changjing and Wang have obtained the displacement solutions of a T-beam resting on an elastic half-space subjected to a uniformly moving load by introducing the equivalent stiffness of the half space interacting with the beam[2]. Li, Fu-leSun and Zhi-zhong have developed a finite difference scheme by the method of reduction of order and shown by the discrete energy method that the scheme is uniquely solvable, unconditionally convergent and stable[3].

Zhang and Chun-guowe have considered the system of nonhomogeneous undamped Timoshenko beam having both ends free with some sufficient conditions and some necessary conditions for the system to have exponential stability based on the operator semigroup technique, the multiplier technique, and the contradiction argument of the frequency domain method[4].A microstructure-dependent model is developed for the Timoshenko beam by using a modified couple stress theory andHamilton's principle. The new model contains a material length scale parameter to account for the microstructural effect, unlike the classical Timoshenko beam theory. The inclusion of this additional material constant enables the new model to capture the size effect. In addition, both bending and axial deformations are considered, and Poisson's effect is included in the current variational formulation, which are different from existing Timoshenko beam models. When the microstructural and Poisson effects are both neglected, the new model recovers the classical Timoshenko beam model[5].

Yesilce, Yusuf Demirdag and OktayIn have obtained the frequency values and mode shapes for freevibration of the multi-span Timoshenko beam subjected to a constant axial compressive force with multiple spring-mass systems for different number of spans and spring- masses with different locations and for different values of axial force[6].

Dong, S B Alpdogan, CTaciroglu and E have worked on three-dimensional information and numerical data to clarify many issues on shear correction factors of Timoshenko beam theory. The displacements (in the SAFE formulation) were vital to the visualization of transverse shear ef- fects in beams of various cross-sections[9].W. Q. Chen et al worked on exact three-dimensional elasticity solutions for FGM thick plates resting on a Winkler–Paster- nak elastic foundation, using the state space method and found that the effects of foundation stiffness on mechanical responses of the plate are considerably different, especially for the strongly thick plates, and that, for a given FGM thick plate, the mechanical behavior of the plate with the softer surface supported by elastic foundation differ significantly from that of the plate with the harder surface subjected to the same foundation[7].Ying, J Lü, CF Chen, WQ investigated two-dimensional elasticity solutions for bending andfree vibration of functionally graded beams resting on Winkler–Pasternak elastic foundations and adopted trigonometric series for the fully simply supported beams to translate the partial differential state equation into an ordinary one, thus making exact solutions possible[8].

Though the literatures on static and dynamic stability of isotropic beams are plenty, the literature on functionally graded beams on elastic foundations reported very less to the best of the authors' knowledge. In the present article, FGO beamshinged at both the ends and resting on parabolic elastic foundation is considered for dynamic stability analysis. This document is a template. An electronic copy can bedownloaded from the conference website. For questions onpaper guidelines, please contact the conference publicationscommittee as indicated on the conference website. Information about final paper submission is available from the conference website.

II. PROCEDURE

A functionally graded sigmoid beam with steel and aluminium as its constituent phases is considered for analysis as shown in Fig.1(a). The beam, hinged at both the endsis subjected to a dynamic axial load $P(t) = P_s + P_d \cos \Omega t$. Where, t is time, P_s is the static component, P_d is the amplitude of the dynamic component and Ω is the frequency of the applied dynamic load component of $P(t)$. The mid-longitudinal(*x-y*) plane is chosen as the reference plane for expressing the displacements as shown in fig. 1(b). The thickness coordinate is measured as *z* from the reference plane. The axial displacement, the transverse displacement, and the rotation of the cross-section are u, w and φ respectively. Fig. 1(c) shows a two nodded beam finite element having three degrees of freedom per node.

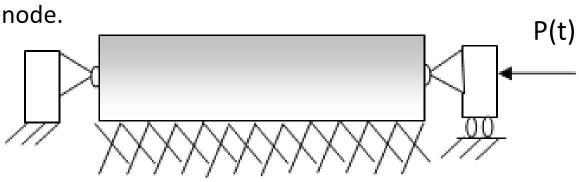

Fig. 1(a) Functionally graded beam subjected to dynamic axial load.

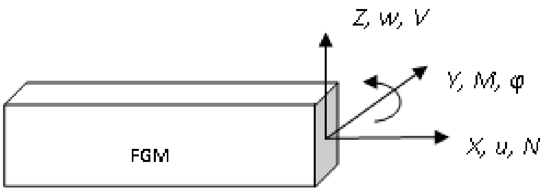

Fig. 1(b) The coordinate system with generalized forces and displacements for the FGSW beam element.

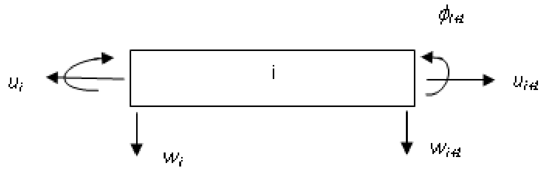

Fig. 1(c) Beam element showing generalized degrees of freedom for i^{th} element.

The generalized displacement vector of the element can be given as

$$\{\hat{u}\} = [u_i \quad w_i \quad \varphi_i \quad u_{i+1} \quad w_{i+1} \quad \varphi_{i+1}] \tag{2.1}$$

The equation of motion for the element subjected to axial force $P(t)$ can be expressed in terms of nodal degrees of freedom as

$$[m]\{\ddot{\hat{u}}\} + [[k_{ef}] - P(t)[k_g]]\{\hat{u}\} = 0 \tag{2.2}$$

The axial load $P(t)$ is taken as

$$P(t) = \alpha P^{\bullet} + \beta_d P^{\bullet} \cos\Omega t,$$

so that, $P_s = \alpha P^{\bullet}$ and $P_d = \beta_d P^{\bullet}$. $P^{\bullet}$ is the critical buckling load of an isotropic beam with similar geometrical dimensions and end conditions and α, β_d are called static and dynamic load factors respectively[10]. Consideringthe application of static and dynamic component of load in the same manner we have eq.(2.2) in the following form

$$[m]\{\ddot{\hat{u}}\} + [[k_{ef}] - P^{\bullet}(\alpha + \beta_d \cos\Omega t)[k_g]]\{\hat{u}\} = 0 \tag{2.3}$$

$$[k_{ef}] = [k_e] + [k_f]$$

where, $[k_e]$, $[k_f]$, $[m]$ and $[k_g]$ are element elastic stiffness matrix, foundation stiffness matrix, mass matrix and geometric stiffness matrix respectively. Assembling the element matrices as used in eq. (2.3), the equation in global matrix form which is the equation of motion for the straight beam, can be expressed as

$$[M]\{\ddot{\hat{U}}\} + [[K_{ef}] - P^{\bullet}(\alpha + \beta_d \cos\Omega t)[K_g]]\{\hat{U}\} = 0 \tag{2.4}$$

$$[K_{ef}] = [K_e] + [K_f]$$

$[M]$, $[K_e]$, $[K_f]$, $[K_g]$ are global mass, elastic stiffness, foundation stiffness and geometric stiffness matrices respectively and $[\hat{U}]$ is global displacement vector. Equation (2.4) represents a system of second order differential equations with periodic coefficients of the Mathieu-Hill type. The periodic solutions for the boundary between the dynamic stability and instability zones can be obtained from Floquet Theory (BOLOTIN (1964)) as follows. A solution with twice the time period which is of practical importance is represented by

$$\hat{U}(t) = c_1 \sin\frac{\Omega t}{2} + d_1 \cos\frac{\Omega t}{2}, \tag{2.5}$$

considering first order expansion.

Substituting eq. (2.5) into eq. (2.4) and comparing the coefficients of $\sin\frac{\Omega t}{2}$ and $\cos\frac{\Omega t}{2}$ terms the condition for existence of these boundary solutions with twice the time period is given by

$$\left([K_{ef}] - (\alpha \pm \beta_d/2)P^{\bullet}[K_g] - \frac{\Omega^2}{4}[M]\right)\{\hat{U}\} = 0 \tag{2.6}$$

Equation (2.6) represents an eigenvalue problem for known values of α, β_d, and $P^{\bullet}$. This equation gives two sets of eigenvalues (Ω) binding the regions of instability due to the presence of plus and minus sign. The instability boundary can be determined from the solution of the equation

$$\left|[K_{ef}] - (\alpha \pm \beta_d/2)P^{\bullet}[K_g] - \frac{\Omega^2}{4}[M]\right| = 0 \tag{2.7}$$

2.1 Free vibration

When α =0, β_d =0, and $\omega = \dfrac{\Omega}{2}$, eq. (2.7) is reduced to a problem of free vibration as

$$\left|[K_{ef}] - \omega^2[M]\right| = 0 \tag{2.8}$$

The solution of eq. (2.8) gives the value of natural frequencies $\{\omega\}$

2.2 Static stability

When α =1, β_d =0, and $\omega = 0$, eq. (2.7) is reduced to the problem of static stability as

$$\left|\left[K_{ef}\right]-P^{\bullet}\left[K_{g}\right]\right|=0 \quad (2.9)$$

The solution of eq. (2.9) gives the values of buckling loads.

2.3 Regions of instability:

ω_1, the fundamental natural frequency and $P^{\bullet}$ the critical buckling load of an isotropic beam with same geometrical dimensions and end conditions are calculated from eq. (2.8) and eq. (2.9) respectively.

Choosing $\Omega=\left(\frac{\Omega}{\omega_1}\right)\omega_1$, eq. (2.7) can be rewritten as

$$\left|\left[K_{ef}\right]-(\alpha\pm\beta_d/2)P^{\bullet}\left[K_g\right]-\left(\frac{\Omega}{\omega_1}\right)^2\frac{\omega_1^2}{4}[M]\right|=0 \quad (2.10)$$

The solution of eq. (2.10) will give two sets of values of $\left(\frac{\Omega}{\omega_1}\right)$ forgiven values of α, β_d, $P^{\bullet}$, and ω_1. The plot between β_d and $\left(\frac{\Omega}{\omega_1}\right)$ will give the regions of dynamic instability.

3 Element matrices

The element matrices for the SFG beam element are derived following the procedure as proposed by Chakraborty et al (2003).

3.1 Shape functions:

The displacement fields according to first order shear deformation beam theory is expressed as

$$U(x,y,z,t)=u(x,t)-z\phi(x,t),$$
$$W(x,y,z,t)=w(x,t), \quad (3.1)$$

The cross-sections are assumed to remain plane after the deformation.

The longitudinal and shear strains are

$$\varepsilon_{xx}=\frac{\partial u}{\partial x}-z\frac{\partial\varphi}{\partial x},\ \gamma_{xz}=-\varphi+\frac{\partial w}{\partial x} \quad (3.2)$$

where $\frac{\partial w}{\partial x}$ is the slope of the deformed longitudinal axis . The stress-strain relation in matrix form can be given by

$$\{\sigma\}=\begin{Bmatrix}\sigma_{xx}\\ \tau_{xz}\end{Bmatrix}=\begin{bmatrix}E(z) & 0\\ 0 & kG(z)\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \gamma_{xz}\end{Bmatrix} \quad (3.3)$$

Where σ_{xx} is the normal stress in longitudinal direction and τ_{xz} is shear stress in $x-z$ plane, $E(z)$ is Young's modulus and $G(z)$ is shear modulus and k is shear correction factor.

The material properties of the FGM that varies along the thickness of the beam are assumed to follow sigmoid distribution given by

$$R(z)=R_t g_1(z)+R_b(1-g_1(z)) \qquad 0\le z\le h/2, \quad (3.4)$$

$$g_1(z)=1-\frac{1}{2}\left(1-\frac{2z}{h}\right)^n \qquad g_2(z)=\frac{1}{2}\left(1+\frac{2z}{h}\right)^n \quad (3.5)$$

where, $R(z)$ denotes a material property such as, E, G, ρ etc., R_t and R_b denote the values of the properties at topmost and bottommost layer of the beam respectively, and n is an index.

The kinetic energy T and elastic strain energy S of an element are given respectively as

$$T=\frac{1}{2}\int_0^l\int_A\rho(z)\left[\left(\frac{\partial U}{\partial t}\right)^2+\left(\frac{\partial W}{\partial t}\right)^2\right]dAdx \quad (3.6)$$

$$S=\frac{1}{2}\int_0^l\int_A E(z)\left[\left(\frac{\partial u}{\partial x}\right)^2+z^2\left(\frac{\partial\varphi}{\partial x}\right)^2-2z\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial\varphi}{\partial x}\right)\right]dAdx$$

$$+\frac{1}{2}\int_0^l\int_A G(z)\left[\varphi^2+\left(\frac{\partial w}{\partial x}\right)^2-2\varphi\frac{\partial w}{\partial x}\right]dAdx \quad (3.7)$$

The governing differential equations can be derived by applying Hamilton's principle as presented below.

$$\frac{\partial(T-S)}{\partial u}=0,\ \frac{\partial(T-S)}{\partial w}=0,\ \text{and}\ \frac{\partial(T-S)}{\partial\varphi}=0 \quad (3.8)$$

The shape functions for the displacement field for finite element formulation are obtained by solving the static part of the eq. (3.8) with the following consideration.

$$u=a_1+a_2x+a_3x^2,$$
$$w=a_4+a_5x+a_6x^2+a_7x^3, \quad (3.9)$$
$$\varphi=a_8+a_9x+a_{10}x^2.$$

The above displacement fields are substituted in the static part of Eq.(3.8) in order to find out the constants of polynomials. Subsequently, the displacement fields are expressed in terms of the nodal degree of freedoms as follows.

$$\{\bar{u}\}=[u\ w\ \varphi]^T=[\aleph(x)]\{\hat{u}\} \quad (3.10)$$

$\aleph(x)$, a 3x6 matrix is the required shape function.

Better convergence can be achieved as the shape functions are obtained from the exact solution of static part of the governing differential equation. Now the shape function can also be expressed as

$$\aleph(x)=\left[\aleph_u(x)\ \ \aleph_w(x)\ \ \aleph_\varphi(x)\right]^T \quad (3.11)$$

where, $\aleph_u(x)$, $\aleph_w(x)$, $\aleph_\varphi(x)$ are

the shape functions for the axial, transverse and rotational degree of freedom respectively.

3.2 Element elastic stiffness matrix

The general force boundary conditions for the element can be given as

$$N_x=\int_A \sigma_{xx}dA=A_{11}\frac{\partial u}{\partial x}-B_{11}\frac{\partial \varphi}{\partial x}$$

$$V_x=\int_A \tau_{xz}dA=A_{55}\left(\frac{\partial w}{\partial x}-\varphi\right)$$

$$M_x=-\int_A z\sigma_{xx}dA=-B_{11}\frac{\partial u}{\partial x}+D_{11}\frac{\partial \varphi}{\partial x} \quad (3.12)$$

where, N_x, V_x, M_x are axial force, shear force and bending moment respectively acting at the boundary nodes.Similarly substituting eq. (3.10) into eq. (3.12) we get

$$[k_e]\{\hat{u}\}=\{F\} \quad (3.13)$$

Where

$\{F\}=[-N_x(0)\ \ -V_x(0)\ \ -M_x(0)\ \ N_x(l)\ \ V_x(l)\ \ M_x(l)]^T$ is the nodal load vector and

$[k_e]$ is the required element elastic stiffness matrix.

3.3 Element elastic foundation matrix:

The work done by the foundation is given by the expression

$$W_f=\frac{1}{2}\int_0^l k_v(x)w^2dx$$

$$=\frac{1}{2}\int_0^l \{\hat{u}\}^T k_v(x)[\aleph_w]^T[\aleph_w]\{\hat{u}\}dx$$

$$=\frac{1}{2}\{\hat{u}\}^T[k_f]\{\hat{u}\} \quad (3.14)$$

Where, k_v is the foundation stiffness parameter per unit width of the beam and $[k_f]=\int_0^l k_v(x)[\aleph_w]^T[\aleph_w]dx$ is element foundation stiffness matrix.

The foundation stiffness that varies along the length of beam considered in present study is as follows.

$k_v=k_o(1-\xi x^2)$ for parabolic variation of foundation stiffness. The foundation modulus is used for analysis purposes. $K=k_oL^4/EI$. The foundation modulus is calculated using an identical steel beam.

3.4 Element geometric stiffness matrix

When an axial load P is applied on the beam element, the work done by the load can be expressed as

$$W_p=\frac{1}{2}\int_0^l P(t)\left(\frac{\partial w}{\partial x}\right)^2 dx \quad (3.15)$$

Substituting the value of w from eq. (3.10) into eq. (3.15) the work done can be expressed as

$$W_p=\frac{P(t)}{2}\int_0^l \{\hat{u}\}^T[\aleph'_w]^T[\aleph'_w]\{\hat{u}\}dx$$

$$=\frac{P(t)}{2}\{\hat{u}\}[k_g]\{\hat{u}\} \quad (3.16)$$

where, $[k_g]=\int_0^l[\aleph'_w]^T[\aleph'_w]dx$ is called the element geometric stiffness matrix.

III. RESULTS AND DISCUSSION

The simulation is carried out for a sigmoid functionally graded (SFG) beam for simply supportedend conditions. The constituent phases chosen are steel and aluminum. The fundamental frequency and critical buckling load are calculated from eq.(2.8) and eq.(2.9) respectively for an isotropic beam made of steel and used to find the parametric instability regions. The calculated values are as follows:

$$\omega_1=6725\,rad/s \qquad P^{\bullet}=11.33x10^7$$

Thebeam with steel-rich bottom is considered for analysis of dynamic stability. The length of the beam is 0.5m, width is 0.1 m with various thicknesses. The material properties are as follows.

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Steel: E =2.1x10^{11} Pa, G =0.8x10^{11} Pa ρ =7.85x10^{3}kg/m^{3}. The shear correction factor k = 0.8667.

Aluminium: E =0.7x10^{11} Pa, G =0.2697x10^{11} Pa, ρ =2.707x10^{3}kg/m^{3}.

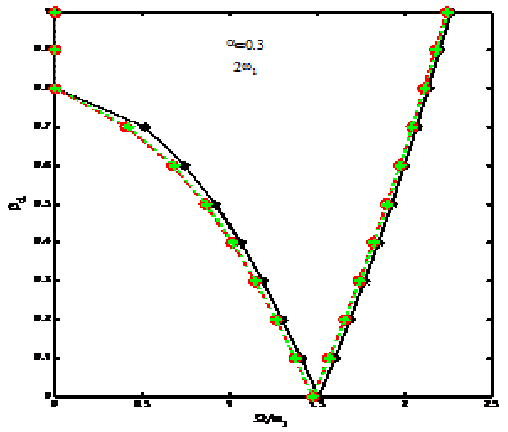

Fig. 2 Effect of material properties on parametric instability of SFG beam resting on parabolic elastic foundation.

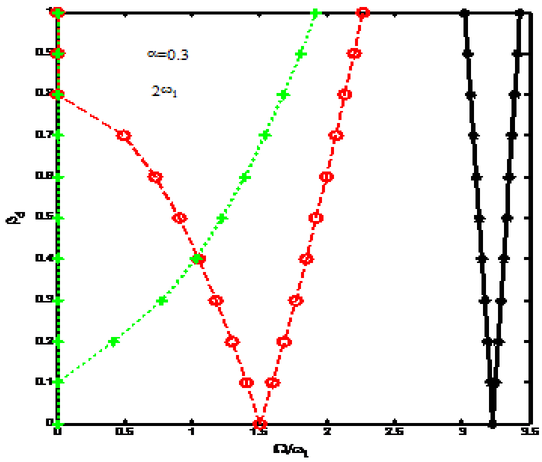

Fig. 3 Effect of beam geometry on parametric instability of SFG beam resting on parabolic elastic foundation.

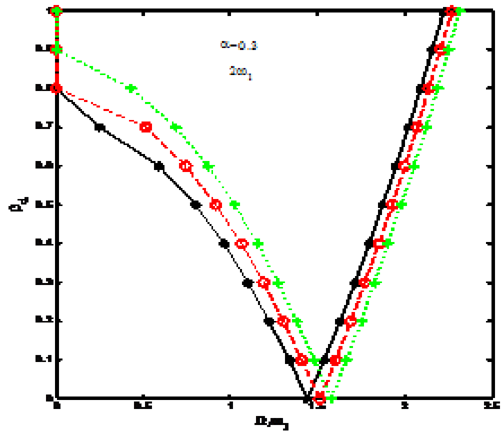

Fig. 4 Effect of foundation on parametric instability of SFG beam resting on parabolic elastic foundation.

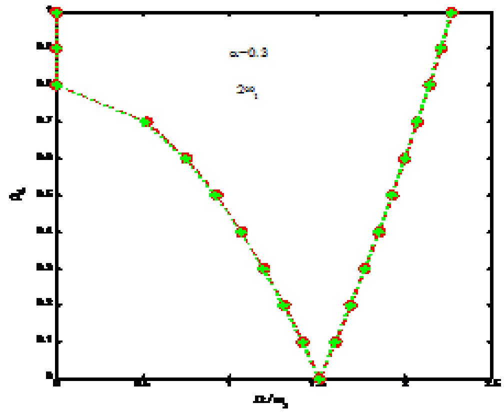

Fig. 5 Effect of foundation parameter on parametric instability of SFG beam resting on parabolic elastic foundation.

Fig. 2 shows the effect of material properties on dynamic instability region for principal mode of beam of thickness 0.0625m with steel-rich bottom.The foundation parameter and foundation modulus are chosen as 0.4 and 500 respectively. As the power index increases the instability region shifts towards the dynamic load factor axis and its area gets widened as well that implies enhancement of instability of the beam. The effect of beam geometry on the dynamic instability of the beam is investigated and depicted in fig. 3. The instability region for principal mode is found to be shifted towards ordinate. The area of instability region increases noticeably with increase of slenderness parameter. Therefore the increase in slenderness parameter makes the beam more prone to parametric instability. The effect of parabolic foundation on dynamic stability is investigated and shown in fig. 4. As expected the beam exhibits better stability with increase in foundation modulus. Fig. 5 shows the effect of foundation parameter on dynamic stability of the beam. The beam becomes slightly more prone to instability as the foundation parameter increases.

IV. CONCLUSIONS

Finite element method is used to investigate parametric instability of SFGbeam resting on parabolic elastic foundation. The following conclusions may be drawn from the above analysis:

1. Foundation improves the dynamic stability behavior of the beam.
2. Sigmoid distribution for designing FGM beams ensures enhancement in instability behavior with increase of power index.

REFERENCES

[1]. T. Kocatu, "Dynamic analysis of eccentrically prestressed viscoelastic Timoshenko beams under a moving harmonic load," vol. 84, pp. 2113–2127, 2006.

[2]. C. Yunmin and W. Changjing, "Steady-state response of a timoshenko beam on an elastic half-space under a moving load," *Acta Mech. Solida Sin.*, vol. 19, no. 1, pp. 26–39, 2006.

[3]. F. Li and Z. Sun, "A finite difference scheme for solving the Timoshenko beam equations with boundary feedback " vol. 200, no. 10471023, pp. 606–627, 2007.

[4]. C. Zhang, "Boundary feedback stabilization of the undamped Timoshenko beam with both ends free," vol. 326, no. 10501039, pp. 488–499, 2007.

[5]. H. M. Ma, X. Ã. Gao, and J. N. Reddy, "Journal of the Mechanics and Physics of Solids A microstructure-dependent Timoshenko beam model based on a modified couple stress theory," vol. 56, pp. 3379–3391, 2008.

[6]. Y. Yesilce and O. Demirdag, "International Journal of Mechanical Sciences Effect of axial force on free vibration of Timoshenko multi-span beam carrying multiple spring-mass systems," vol. 50, pp. 995–1003, 2008.

[7]. Z. Huang, C. Lü, and W. Chen, "Benchmark solutions for functionally graded thick plates resting on Winkler–Pasternak elastic foundations," *Compos. St*

[8]. J. Ying, C. Lü, and W. Chen, "Two-dimensional elasticity solutions for functionally graded beams resting on elastic foundations," *Compos. Struct.*, 2008.

[9]. S. B. Dong, C. Alpdogan, and E. Taciroglu, "International Journal of Solids and Structures Much ado about shear correction factors in Timoshenko beam theory," *Int. J. Solids Struct.*, vol. 47, no. 13, pp. 1651–1665, 2010.

[10]. S. Bose, P. Chugh, and A. Gupta, "Effect of Elastic Foundation & Damping on Parametric Instability of Beams," 2012.

[11]. W. S. E. Foundation, "Journal of Engineering Research and Studies E-ISSN 0976-7916," no. Ii, 2010.