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β_2 NEAR SUBTRACTION SEMIGROUP

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ABSTRACT
In this paper we introduce the notation β_2 near subtraction semigroup and study some of their properties.

Key words: Completely semiprime, ideal, regular.

1. Introduction

B.M.Schein [7] considered systems of the form (X; o;/), where X is a set of functions closed under the composition "o" of functions (and hence (X; o) is a function semigroup) and the set theoretic subtraction "/" (and hence (X;/) is a subtraction algebra in the sense of [1]). He proved that every subtraction semigroup is isomorphic to a difference semigroup of invertible functions. B.Zelinka [10] discussed a problem proposed by B.M.Schein concerning the structure of multiplication in a subtraction semigroup. He solved the problem for subtraction algebras of a special type, called the atomic subtraction algebras. For basic definition one may refer to Pilz [6]. In near rings the notation of β_2 introduced by Sugantha et al [8]. Motivated by this concept, we introduced $\beta_2$nearsubtraction semigroups. (i.e.,) Let X be a right near subtraction semigroup. If for every x,y in X, xXy = xyX we say β_2 near subtraction semigroup. Also we distinguish them by characterizing separately. Throughout this paper X stands for a right near subtraction semigroup with atleast two elements.

2. PRELIMINARY CONCEPTS AND RESULTS

Definition:2.1 A nonempty set X together with binary operations **"–"** and is said to be subtraction algebra if it satisfies the following:

(i) $x-(y-x)=x$.
(ii)$x-(x-y)=y-(y-x)$.
(iii)$(x-y)-z=(x-z)-y$,for every $x,y,z\in X$.

Definition:2.2 A nonempty set X together with two binary operations **"–"** and **"•"** is said to be a subtraction semigroup if it satisfies the following:

(i) (X,−)is a subtraction algebra.
(ii) (X,•)isasemigroup.
(iii) $x(y-z)=xy-xz$ and $(x-y)z=xz-yz$, for every $x,y,z\in X$.

Definition:2.3 A non empty set X together with two binary operations **"–"** and**'•"**is said to be a near subtraction semigroup (right) if it satisfies the following:

(i) (X,−)isasubtractionalgebra.
(ii) (X,•)isasemigroup.
(iii) $(x-y)z=xz-yz$,foreveryx,y,$z\in X$.

Remark:2.4 The symbol X stands for a near subtraction semigroup (X,–,•) with at least two elements. We write xy for x.y for any two elements x, y of X. It is clear that $0.x= 0$,for every $x\in X$. It can be easily proved that $x-0=x$ and $0-x=0$, for all $x\in X$.

Definition:2.5

(i) $X_0 = \{n\in X \mid n0 = 0\}$ is called the zero-symmetric partof X.
(ii) $X_c=\{n\in X \mid n0 = n\} = \{n\in X \mid nn' = n, \text{ for all } n'\in X\}$ is called the constant part of X.
(iii) X is called zero-symmetric, if $X = X_0$.
(iv) X is called constant, if $X = X_c$.
(v) $X_d = \{n\in X \mid n(x-y) = nx-ny, \text{ for all } x, y \text{ in } X\}$ is the set of all distributive elements of X.
(vi) A near subtraction semigroup X is called distributive, if $X = X_d$.

Notations: 2.6

(1) E denotes the set of all idempotent of X.

(2) L denotes the set of all nilpotent elements of X.

(3) If A is any non empty subset of X, then $A^* = A-\{0\}$.

(4) C(X) denotes the centre of X.

(5) C(a) = {n∈X **/** an = na}.

(6) X* = X–{0}

Definition:2.7 An element e∈X is said to be idempotent if $e^2 = e$.

Definition:2.8 An element a∈X is said to be central if ax = xa.

Definition: 2.9 An element x∈X is said to be nilpotent if there exists positive integer n such that $x^n=0$.

Definition:2.10 A near subtraction semigroup X is said to be (∗,IFP) provided for all a,b,n in X, if ab = 0 then anb = 0.

Notation:2.11 If A and B are any two subsets of X, then AB = {ab / a∈A and b∈B} and A∗B = {a(a′–b) –aa′ / a, a′∈A and b∈B}.

Definition:2.12 A nonempty subset S of a subtraction semigroup X is said to be a subalgebra of X, if x–x′∈S whenever x,x′∈S.

Definition:2.13 A near subtraction semigroup X is said to be property p_4 if for all ideals I of X and for all x,y in X

Definition:2.14 A subtraction semigroup X is said to beIFP (intersection of factors property) if for a,b in X if ab = 0 implies axb = 0, for all x∈X.

Result:2.15 A near subtraction semigroup X has no non-zero nilpotent elements if and only if $x^2 = 0 \Rightarrow x = 0$, for all x in X.

Definition: 2.16 If X satisfies **(i)** xy = 0 ⇒yx = 0, for all x,y in X **(ii)** X has IFP then X is said to have (∗, IFP).

Definition:2.17 A near subtraction semigroup X is regular if for every x in X there is some y in X such that x = xyx.

Definition:2.18 A nonempty subset I of X is called

(i) A left ideal of X if x-y∈ I andx∈I and y∈X and XI⊆ I

(ii) A right ideal of X if x-y∈ I andx∈I and y∈X and IX⊆ I.

(iii) An ideal of X if I is both left and right ideal of X.

Definition:2.19 Let P be an ideal of X. P is called

(i) a prime deal if for all ideal I,J of X, IJ⊆P then I⊆ P or J⊆P.

(ii) a completely prime ideal , if for any ab in X, ab∈P then either a∈P or b∈P.

3. β_2 near subtraction semigroup

In this section we define β_2 near subtraction semigroup and give certain examples of these new concepts.

Definition:3.1. Let X be a right near subtraction semigroup. If for every x,y in X, xXy = xyX we say β_2 near subtraction semigroup.

Example:3.1.1 Let X={0,a,b,c} in which '–' and '•' is defined as follows

–	0	a	b	1
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
1	1	b	a	0

•	0	a	b	1
0	0	0	0	0
a	0	a	b	1
b	0	0	0	0
1	0	a	b	1

Obviously (X,–,•) is a β_2 near subtraction semigroup.

Theorem:3.2 Let X be a β_2 near subtraction semigroup. If X has identity 1, then X is zero symmetric.

Proof: Let X be a β_2 near subtraction semigroup. Then for all x,y in X, x.Xy = xyX Putting x = 1, we get 1X y = 1yX , for all y in X . (ie) Xy = yX , for all y in X . When y = 0, X 0 = 0X = {0}. It follows that X is zero-symmetric.

Remark:3.2.1 The converse of above theorem is not valid.

Example:3.2.2 Let X={0,a,b,c} in which '–' and '•' is defined as follows

–	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	b	a	0

•	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	0	0
c	0	a	0	a

This is a zero-symmetric β_2 near subtraction semigroupBut it has no identity.

Theorem:3.3 If X is a β_2 near subtraction semigroupxXx= x^2X, for all xin X.

Proof: When X is a β_2 near subtraction semigroup xXy = xyX-------------(∗). The results follows by replacing y by x in equation (∗).

Remark:3.3.1 The converse of the above theorem is not true.

Example:3.3.2 Let X={0,a,b,c} in which '–' and '•' is defined as follows

−	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	b	a	0

•	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	a	b	c
c	a	0	c	b

The near subtraction semigroup satisfies the condition $xXx = x^2 X$ for all x in N. But it is not β_2 near subtraction semigroup. $yXx \neq yxX$.

Theorem : 3.4 Let X be a zero-symmetric, β_2 near subtraction semigroup with regular. Then we have

(i) $L = \{ 0 \}$

(ii) X has {* , IFP}

(iii) $E \subseteq C(X)$ if $E \subseteq X_d$

(iv) Any ideal of X is completely semiprime.

(v) X has property P_4.

Proof: (i) Since X is regular, By theorem 3.3 demands that $x \in xXx = x^2 X$, for all x in X. Therefore $x = x^2 n$, for all n in X. Suppose $x^2 = 0$ .Clearly x = 0. Hence L = {0}

(ii) By (i), L = {0}. Since X is zero-symmetric, X has {*, IFP}.

(iii) Let $e \in E$, since X is β_2 near subtraction semigroup. eXe = eeX = eX. Therefore for any x in X, ene = eu and en = eve, for some u, v in X.

Now ene = evee. Thus ene = eve = en, for all n in X ---------(1)

Since $E \subseteq X_d$ and $X = X_0$, e(ne – ene) = 0 and ne (ne – ene) = n0 = 0.

This implies that ene (ne – ene) = 0. ne (ne – ene) = e ne (ne – ene) = 0

Consequently $(ne - ene)^2 = 0$.

(i) guarantees that (ne – ene) = 0 ----------(2)

Therefore ene = ne, for all n in X .From equation (1) and (2), we get ne = ene, for all n in X . Thus $E \subseteq C(X)$

(iv) Let I be an ideal of X. Then $IX \subseteq I$ ----------- (3) and let $a^2 \in I$, for some a in X . Since X is regular, for all a in X. a = axa. Then $a \in aXa$. Since X is a β_2 near subtraction semigroup, $a \in a^2 X$.

But $a^2 X \in IX \subset I$ (by equation (3)). Therefore $a \in I$. Consequently I is completely semi prime.

(v) Let I be an ideal of X. Then $IX \subseteq I$ --------- (4)

Since X is zero-symmetric, $XI \subseteq I$ -------- (5)

Let $x\, y \in I$. Now $(yx)^2 = (yx)y(x) = y(xy)\, x \in XIX \subseteq I$ (By equation (5)).

By equation(4), $(yx)^2 \in I$ .Using (iv), we get $yx \in I$.Consequently X has property P_4.

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