

**DESIGN & ANALYSIS OF LATERALLY LOADED PILE USING FINITE DIFFERENCE METHOD****SHAIK SHAHABAZBANU¹, P.NAGARAJAN², N.SIVASHANKAR REDDY³**¹M.Tech Structural Engineering & Sir Vishveshwaraiah Institute of science & Technology, AP² Assistant Professor, Dept. of Civil Engineering, Sir Vishveshwaraiah Institute of science & Technology, AP³ Associate Professor, Dept. of Civil Engineering, Sir Vishveshwaraiah Institute of Science & Technology-AP**ABSTRACT**

The main aim of the project is to calculate structural parameters like Deflection(Y), Moment (M), Shear(S), Soil reaction(P) etc., at each and every point of the pile. When the soil near the ground surface is not capable of supporting a structure, deep foundations are required to transfer the loads to deeper strata.

Deep foundations are used when the soil is very weak below the ground surface. The most common types of deep foundations are piles, piers and caissons. A pile subjected to lateral loading is one of the class of problems that involve interaction of soil and structures.

The main purpose of implementing pile is to carry vertical loads when the soil is weak. When the pile is subjected to lateral loading a solution cannot be obtained without accounting for the deformation of both pile and soil. Pile foundations are used to resist the horizontal forces and the vertical loads.

Pile foundation is used to reduce differential settlements. The deflection of the pile and the lateral resistance of soil are interdependent. Therefore iterative techniques are almost necessary to achieve a solution for particular case of loading on the pile.

In this project we are using Finite Difference Method (F.D.M). This method is applicable for regular shapes. In this project we are taking ALUMINIUM and PVC as pile materials. Finally structural parameter values are compared with C program or mat lab using matrix inversion method and also manually by using MS excel.

Keywords: Deflection, Moment, Shear & FDM.

1. INTRODUCTION**1.1 General**

Deep foundations are employed principally when weak or otherwise unsuitable soil exists near the ground surface and vertical loads must be carried to strong solids at depth. Deep foundations have a number of other uses, such as to resist scour; to sustain axial loading by side resistance in strata of granular soil or competent clay; to allow above-water construction when piles are driven through the legs of a template to support an offshore platform; to serve as breasting and

morning dolphins; to improve the stability of slopes; and for a number of other special purposes. A pile subjected to lateral loading is one of a class of problems that involve the interaction of soils and structures. Soil – structure interaction is encountered in every problem in foundation engineering, but in some cases the structure is so stiff that a solution can be developed assuming nonlinear behaviour for the soil and no change in shape for the structural unit.

Engineers understood early that a pile under lateral load would act as a beam. Hetenyi

(1946) wrote a book giving the solution of the differential equation for a beam on a foundation, with a linear relationship between pile deflection and soil response. In the early 1950s, Shell Oil Company was planning to install an offshore platform at Block 42 in 25m of water near the Louisiana coast, where the soil was predominantly soft clay. The procedure was available for computing lateral forces on the platform during a hurricane.

1.2 Finite Element Method

Finite element analysis was developed as a numerical method of stress analysis. But, now it has been extended as a general method of solution to many complex engineering and physical science problems. As it involves lot of calculations, its growth is closely linked with the development of computer technology.

In Finite Difference Method, the derivatives in the governing partial differential equations are written in the terms of difference equations. In this method, at each of the pivotal point of the intervals, an equation expressing the differential equations by the finite difference can be established. A set of simultaneous equations are developed and they are solved using the boundary conditions at pivotal points. The computations can be easily handled mechanically on calculators or digital computers.

1.3 Boundary Element Method

As the name implies, in this approach the governing differential equations are transformed into integral identities which are applicable over the surface or boundary. These integrals are numerically integrated over the boundary which is divided into small boundary segments (boundary segments). As in the other numerical approaches, provided that the boundary conditions are satisfied, a system of linear algebraic equations emerges for which a unique solution is obtained.

The boundary element method can easily accommodate geometrically complex boundaries. Furthermore, since all the approximations are restricted to the surface, it can model region with rapidly changing variables with better accuracy than the Finite Element Method.

1.4 Discrete Element Method

The distinct element method is conceptually and algorithmically the simplest of all the methods of stress analysis. The distinct element

method is suitable for discontinuous systems such as rocks and soils and the method treats the medium as a discontinue i.e., assembly of quasi rigid blocks interacting through deformable joints of deformable stiffness. This method was originally developed by Cundall & Strack (1979) to model the progressive failure of rock slopes. The distinct element method is normally used to determine if a medium will fail under a given set of applied loads including gravity or to calculate the displacements that are accumulated if the system finally stabilizes. This technique has the advantage that both geometric and material non – linearity's are taken into account and also the numerical divergence is easily recognizable in terms of anomalous physical behavior.

1.5 Advantages of Finite Difference Method

1. Finite Difference Method can be advantageously employed for complex loading, boundary configurations, mathematical expressions, load distribution sectional properties etc...
2. Partial differential equations are invariably required in solving problems of two – dimensional structural elements can be made easier by using FDM.
3. This method is used for determination of moments, shear forces, deflections, buckling of columns which involve the differential equations.
4. FDM is applicable for statically determinate and statically indeterminate structures, vibration problems, torsional problems and beams on elastic foundations.

1.6 Disadvantages of Finite Difference Method

1. Finite Difference Method needs large number of nodes to get good results.
2. With FDM, fairly complicate problems can be handled.
3. FDM is not applicable for irregular shapes.
4. This method does not give values at any point except at nodes.
5. This method makes point wise approximation to the governing equation.
6. It ensures continuity only at node points.

1.7 Formulation of the Equation by FDM of Laterally Loaded Pile

The assumption is made that a bar on an elastic foundation is subjected to horizontal loading

and to a pair of compressive forces P_x acting in the center of gravity of the end cross sections of the bar. If an infinitely small unloaded element, bounded by two horizontals a distance dx apart, is cut out of this bar the equilibrium of moments (ignoring second-order terms) leads to the equation.

$$(M + dM) - M + P_x dy - V_v dx = 0$$

Or

$$\frac{dM}{dx} + P_x \frac{dy}{dx} - V_v = 0$$

Differentiating the above equation with respect to x , the following equation is obtained.

$$\frac{d^2M}{dx^2} + P_x \frac{d^2y}{dx^2} - \frac{dV_v}{dx} = 0.$$

The following identities are noted:

$$\frac{d^2M}{dx^2} = E_p I_p \frac{d^4y}{dx^4}$$

$$\frac{dV_v}{dx} = p$$

And making the indicated substitutions we get generalized governing differential equation for a pile under lateral loading which is as follows.

$$E_p I_p \frac{d^4y}{dx^4} + P_x \frac{d^2y}{dx^2} + p = 0$$

The derivatives of differential equation needed for formulation of finite difference equations are as follows.

y – Deflection

$$\frac{dy}{dx} \text{ – slope (s)}$$

$$EI \frac{d^2y}{dx^2} = \text{moment (M)}$$

$$EI \frac{d^3y}{dx^3} = \text{shear (V)}$$

$$EI \frac{d^4y}{dx^4} = \text{soil reaction (P)}$$

$$\frac{dy}{dx} = \frac{y_{i-1} - y_{i+1}}{2h}$$

$$\frac{d^2y}{dx^2} = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}$$

$$\frac{d^3y}{dx^3} = \frac{y_{i-2} - 2y_{i-1} + 2y_{i+1} - y_{i+2}}{2h^3}$$

$$\frac{d^4y}{dx^4} = \frac{y_{i-2} - 4y_{i-1} + 6y_i - 4y_{i+1} + y_{i+2}}{h^4}$$

1.8 Boundary Conditions

Top pile:

$$y_{-2} = \frac{2P_t h^3}{EI} + \frac{2P_t e h^2}{EI} + 4y_0 - 4y_1 + y_2 \rightarrow (5)$$

Bottom pile:

$$y_{m+2} = y_{m-2} - 4y_{m-1} + 4y_m \rightarrow (7)$$

2. EQUATIONS FOR FREE ENDED PILE

2.1 For Aluminium

Load: 57.01 N

$$S_0 = -0.022445, S_1 = -0.0161019, S_2 = 9.87478 \times 10^{-3},$$

$$S_3 = 0.0176094, S_4 = 0.0237048$$

$$M_0 = 1.99535 \text{ N-M}, M_1 = 0.78929 \text{ N-M}, M_2 = -0.04743 \text{ N-M}$$

$$M_3 = 0.03304 \text{ N-M}, M_4 = 0 \text{ N-M}$$

$$V_0 = 57.01 \text{ N}, V_1 = 54.36 \text{ N}, V_2 = 55.728 \text{ N}$$

$$V_3 = 55.807 \text{ N}, V_4 = 55.790 \text{ N-M}$$

Load: 129.3 N

$$\frac{P_t e h^2}{EI} = 0.24898$$

$$S_0 = -0.050908, S_1 = -0.036521, S_2 = 2.239866 \times 10^{-3},$$

$$S_3 = 0.039939, S_4 = 0.053763192$$

$$M_0 = 4.525 \text{ N-M}, M_1 = 1.78841 \text{ N-M}, M_2 = -0.10731 \text{ N-M}$$

$$M_3 = 0.074875 \text{ N-M}, M_4 = 0 \text{ N-M}$$

$$V_0 = 129.3 \text{ N}, V_1 = 123.30 \text{ N}, V_2 = 126.39 \text{ N}$$

$$V_3 = 126.561 \text{ N}, V_4 = 126.541 \text{ N-M}$$

Load: 191.08 N

$$\frac{P_t e h^2}{EI} = 0.3679, S_0 = -0.075230, S_1 = -0.0539689$$

$$S_2 = 3.309864 \times 10^{-3}, S_3 = 0.059, S_4 = 0.07945228$$

$$M_0 = 6.6878 \text{ N-M}, M_1 = 2.646 \text{ N-M}, M_2 = -0.161176 \text{ N-M}$$

$$M_3 = 0.125199 \text{ N-M}, M_4 = 0 \text{ N-M}$$

$$V_0 = 191.08 \text{ N}, V_1 = 182.221 \text{ N}, V_2 = 186.78 \text{ N}$$

$$V_3 = 187.03, V_4 = 187.00 \text{ N-M}$$

Load: 269.75 N

$$\frac{P_t e h^2}{EI} = 0.4339, S_0 = -0.1099215, S_1 = -0.078697,$$

$$S_2 = 4.768522 \times 10^{-3}, S_3 = 0.085881, S_4 = 0.115619928$$

$$M_0 = 9.4412 \text{ N-M}, M_1 = 3.7356 \text{ N-M}, M_2 = -0.22753 \text{ N-M}$$

$$M_3 = 0.176738 \text{ N-M}, M_4 = 0 \text{ N-M}$$

$$V_0 = 269.75 \text{ N}, V_1 = 257.243 \text{ N}, V_2 = 263.035 \text{ N}$$

$$V_3 = 264.035 \text{ N}, V_4 = 263.990 \text{ N}$$

Load: 401.27 N

$$\frac{P_t e h^2}{EI} = 0.772, S_0 = -0.1580159, S_1 = -0.113357,$$

$$S_2 = 6.95166 \times 10^{-3}, S_3 = 0.1239691, S_4 = 0.16688099$$

$$M_0 = 14.044 \text{ N-M}, M_1 = 5.557 \text{ N-M}, M_2 = -0.3384 \text{ N-M}$$

$$M_3 = 0.26291 \text{ N-M}, M_4 = 0 \text{ N-M}$$

$V_0=401.27N, V_1=400.289N, V_2=402.630N$

$V_3=403.549N, V_4=401.289N$

2.2 For PVC

Load:57.01 N

$S_0= 0.011092, S_1= 2.36957*10^{-3}, S_2= 4.5408*10^{-5},$
 $S_3= 9.0488*10^{-5}, S_4=-1.304*10^{-5}$

$V_0=57.01N, V_1=56.36N, V_2=57.35N$

$V_3=57.38N, V_4=57.37N$

Load:129.3 N

$$\frac{P_t e h^2}{EI} = 7.088$$

$S_0= 0.025148, S_1= 5.3699*10^{-3}$

$S_2= 1.03338*10^{-4}, S_3= 2.0497*10^{-4}$

$S_4=2.96*10^{-5}$

Load:191.08 N

$$\frac{P_t e h^2}{EI} = 10.476$$

$S_0= 0.03718, S_1= 7.940105*10^{-3}$

$S_2= 1.54906*10^{-4}, S_3= 2.67616*10^{-4}$

$S_4= -1.24612*10^{-4}$

Load:269.75 N

$$\frac{P_t e h^2}{EI} = 14.789, S_0= 0.052490$$

$S_1=0.011209, S_2= 2.1867*10^{-3}$

$S_3= 0.377792*10^{-4}, S_4=-1.759*10^{-4}$

Load:401.27 N

$$\frac{P_t e h^2}{EI} = 21.999, S_0= 0.078082$$

$S_1= 0.016674, S_2= 3.25236*10^{-4}, S_3= 5.620*10^{-4}$

$S_4=-2.61664*10^{-4}$

3. GRAPHS BETWEEN DEPTH AND DEFLECTION

3.1 For Aluminium Pile

Table 3.1 Depth and Deflection values for Aluminium

S.N O	Dept h (M)	Deflection (Y)				
		Load 57.01 N	Load 129.3 N	Load 191.08 N	Load 269.7 N	Load 401.2 N
1	0	0.158 7	0.359 9	0.5319	0.750	0.850 9
2	0.25	0.038	0.008 5	0.0126	0.017 8	0.018 4
3	0.5	0.0010	-0.002	-0.003	-0.004	-0.005
4	0.75	0	-10e-4	-0.001	-2e-4	-0.003
5	1	0	0	0	0	0

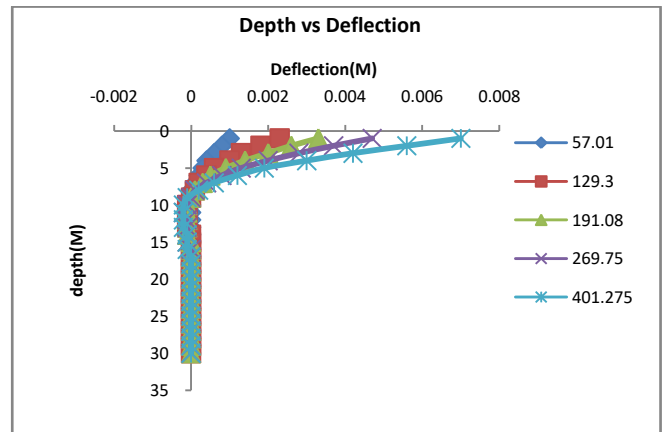


Fig 3.1. Graph between Depth and Deflection for aluminium Pile.

6.2 For PVC Pile

Table 3.2 Depth and Deflection values for PVC

S.N O	Dept h (M)	Deflection (Y)				
		Load 57.01 N	Load 129.3 N	Load 191.08 N	Load 269.7 N	Load 401.2 N
1	0	0.158 9	0.359 9	0.5419	0.750	0.869 1
2	0.25	0.039	0.008 9	0.0126	0.017 8	0.019 4
3	0.5	0.0011	-0.002	-0.003	-0.004	-0.005
4	0.75	0	-10e-4	-0.001	-2e-4	-0.003
5	1	0	0	0	0	0

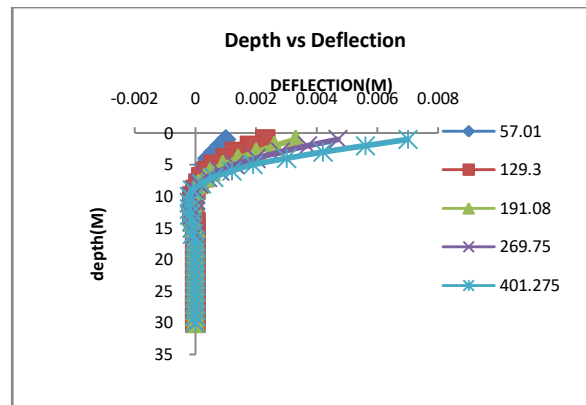


Fig 3.2. Graph between Depth and Deflection for PVC Pile.

6.3 For Aluminium Pile

Table 3.3 Depth and Moment values for Aluminium

S.N O	Dept h (M)	Moment (M) N-M				
		Load 57.01 N	Load 129.3 N	Load 191.08 N	Load 269.7 N	Load 401.2 N
1	0	1.995	4.525	6.687	9.441	14.00
2	0.25	11.11	26.12	39.207	52.61	67.81
3	0.5	0.419	0.958 7	1.404	1.987	2.034
4	0.75	-0.066	-0.14	-0.2214	-0.312	-0.417
5	1	0	0	0	0	0

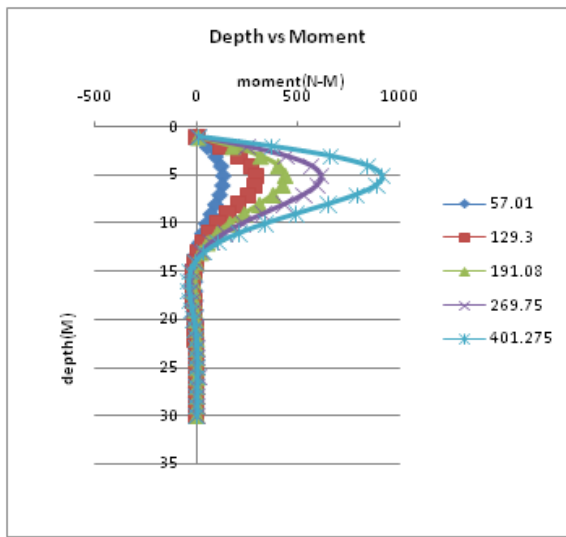


Fig 3.3. Graph between Depth and Moment for aluminium Pile.

Table 3.4 Depth and Deflection values for PVC

S.N O	Dept h (M)	Moment (M) N-M				
		Load 57.01 N	Load 129.3 N	Load 191.08 N	Load 269.7 N	Load 401.2 N
1	0	1.995	4.525	6.687	9.441	14.00
2	0.25	12.11	25.21	39.207	52.61	66.89
3	0.51	0.519	0.965	1.404	1.987	2.034
4	0.75	-0.066	-0.14	-0.2214	-0.312	-0.418
5	1	0	0	0	0	0

		6	0		4	8
3	0.51	55.72	126.3	186.78	263.6	402.0
4	0.75	55.80	126.5	187.03	264.0	403.5
5	1	55.78	12.54	187.03	263.0	401.2

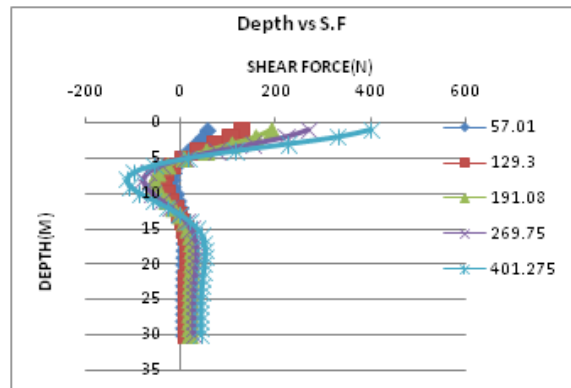


Fig 3.5. Graph between Depth and Shear for Aluminium Pile.

Table 3.6 Depth and Shear values for PVC

S.N O	Dep th (M)	Moment (M) N-M				
		Load 57.0 1N	Load 129. 3N	Load 191.0 8N	Load 269. 7N	Load 401. 2N
1	0	57.0	129.	191.0	269.	401.
		1	3	8	75	27
2	0.25	54.3	123.	184.1	257.	400.
		66	30	4	24	78
3	0.51	55.3	126.	186.7	263.	402.
		8	39	8	08	63
4	0.75	55.8	126.	187.0	264.	403.
		0	51	3	08	54
5	1	55.8	12.7	187.0	263.	401.
		0	8	0	12	28

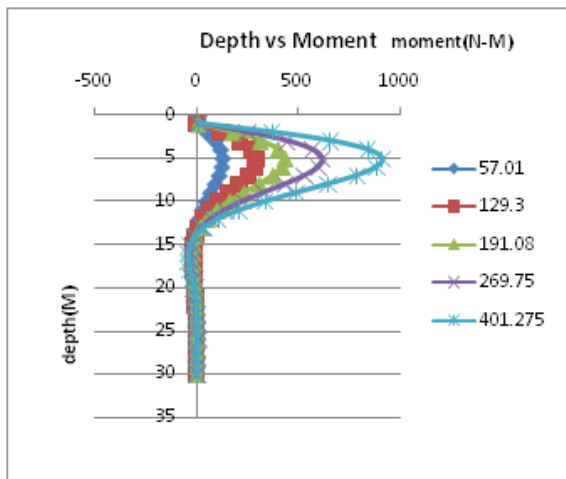


Fig 3.4. Graph between Depth and Moment for PVC Pile.

Table 3.5 Depth and Shear values for Aluminium

S.N O	Dept h (M)	Moment (M) N-M				
		Load 57.01 N	Load 129.3 N	Load 191.08 N	Load 269.7 N	Load 401.2 N
1	0	57.01	129.3	191.08	269.7	401.2
					5	7
2	0.25	54.36	123.3	184.22	257.2	401.2

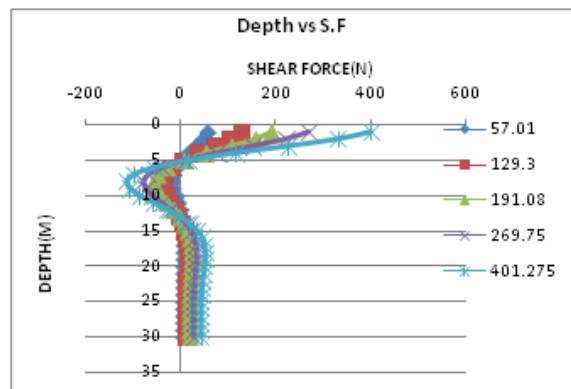


Fig 3.6 Graph between Depth and Shear for PVC Pile.

Table 3.7 Depth and Soil reaction values for Aluminium

S.N	Dept h (M)	Moment (M) N-M				
		Load 57.01 N	Load 129.3 N	Load 191.08 N	Load 269.7 N	Load 401.2 N
1	0	0	0	0	0	0
2	0.25	2.64	5.99	8.859	12.50 6	16.58
3	0.51	-1.36	3.089	-4.56	-6.44	-8.05
4	0.75	-0.073	-0.16	-0.246	-0.34	-0.42
5	1	0.008	0.007	0.0295	0.041	0.06

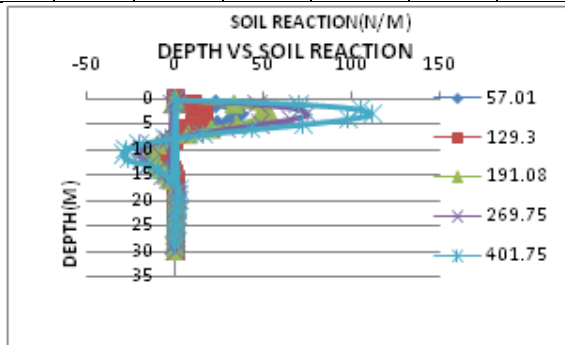


Fig 3.7 Graph between Depth and Soil Reaction for Aluminium Pile

Table 3.8 Depth and Soil reaction values for PVC

S.N	Dept h (M)	Moment (M) N-M				
		Load 57.01 N	Load 129.3 N	Load 191.08 N	Load 269.7 N	Load 401.2 N
1	0	0	0	0	0	0
2	0.25	2.67	5.99	6.68	12.50 6	16.58
3	0.51	-1.36	-3.089	-4.56	-6.44	-8.05
4	0.75	-0.007	-0.16	-0.246	-0.34	-0.42
5	1	0.008	0.007	0.0295	0.041	0.06

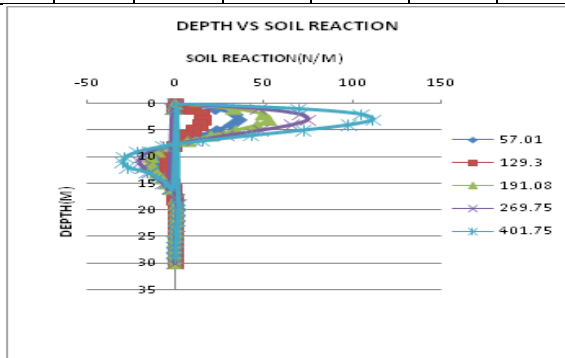


Fig 3.8 Graph between Depth and Soil Reaction for PVC Pile.

4. CONCLUSION

- A pile subjected to lateral loading is one of the class of problems that involve interaction of soil and structures.

- For a pile under lateral loading, a solution cannot be obtained without accounting deformation of both pile and soil. The deflection of the pile and the lateral resistance are interdependent.
- Because of non linearity of soil, sometimes of the pile interactive techniques are always necessary to achieve a solution for a particular case of loading on pile.
- In this project a computer code has been developed for the analysis of laterally loaded pile using finite difference method. First pile is subdivided into m increments in which (m+1) equations can be written leading to (m+5) unknowns.
- Substituting appropriate boundary conditions fictitious nodes at top and bottom portion of pile has been eliminated leading to (m+1) unknowns and (m+1).
- The program are validated for (m+1) equations and (m+1) unknowns and checked for correctness by solving hypothetical numerical problems.
- Finally without conducting any laboratory test for a pile we can calculate any structural parameter at each and every point of an element or structure.

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