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# OBSERVATIONS ON THE HYPERBOLA $y^2 = 30x^2 + 1$

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**ABSTRACT**

The binary quadratic equation $y^2 = 30x^2 + 1$ is considered and a few interesting properties among the solutions are presented. Employing the integral solutions of the equation under consideration, a special pythogorean triangle is obtained.

**Keywords**: binary quadratic, hyperbola, integral solutions, pell equation

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## INTRODUCTION

The binary quadratic equation of the form $y^2 = Dx^2 + 1$, where D is a non-square positive integer, has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values [1,2,3,4]. In [5] infinitely many pythogorean triangles in each of which hypotenuse is four times the product of the generators added with unity are obtained by employing the non-integral solutions of binary quadratic equation $y^2 = 3x^2 + 1$. In [6], a special pythogorean triangle is obtained by employing the integral solutions of $y^2 = 10x^2 + 1$. In [7], different patterns of infinitely many pythogorean triangles are obtained by employing the non-trivial solutions of $y^2 = 12x^2 + 1$. In this context one may also refer [8-14]. These results have motivated us to search for the integral solutions of yet another binary quadratic equation $y^2 = 30x^2 + 1$ representing a hyperbola. A few interesting properties among the solutions are presented. Employing the integral solutions of the equation under consideration, a special pythogorean triangle is obtained.

Notations Used:

$t_{m,n}$ – Polygonal number of rank n with size m.

$P_n^m$ – Pyramidal number of rank n with size m.

$CP_n^m$ – Centered pyramidal number of rank n with size m.

$CP_{m,n}$ – Centered polygonal number of rank n with size m.

$GNO_n$ – Gnomonic number of rank n.

$S_n$ – Star number of rank n.

**METHOD OF ANALYSIS**

The binary quadratic equation representing hyperbola under consideration is

$$y^2 = 30x^2 + 1 \qquad (1)$$

whose general solution $(x_n, y_n)$ is given by $x_n = \frac{g}{2\sqrt{30}}, y_n = \frac{f}{2}$ where

$f = (11 + 2\sqrt{30})^{n+1} + (11 - 2\sqrt{30})^{n+1}$ and

$g = (11 + 2\sqrt{30})^{n+1} - (11 - 2\sqrt{30})^{n+1}$, n = 0, 1, 2, ....

The recurrence relations satisfied by x and y are given by

$$y_{n+2} - 22y_{n+1} + y_n = 0, y_0 = 11, y_1 = 241$$

$$x_{n+2} - 22x_{n+1} + x_n = 0, x_0 = 2, x_1 = 44$$

Some numerical examples of x and y satisfying (1) are given in the following table:

| n | $x_n$ | $y_n$ |
|---|---|---|
| 0 | 2 | 11 |
| 1 | 44 | 241 |
| 2 | 966 | 5291 |
| 3 | 21208 | 116161 |
| 4 | 465610 | 2550251 |
| 5 | 10222212 | 55989361 |
| 6 | 224423054 | 1229215691 |
| 7 | 4927084976 | 26986755841 |

From the above table we observe some interesting properties:

1. $X_n$ is always even.
2. $Y_n$ is always odd.
3. $y_{2n} \equiv 0 mod(11)$.
4. $x_{2n+1} \equiv 0 mod(44)$.

A few interesting properties between the solutions and special numbers are given below:

1. $22y_{2n+2} - 120x_{2n+2} + 2$ is a perfect square.
2. $6(22y_{2n+2} - 120x_{2n+2} + 2)$ is a nasty number.
3. $22y_{3n+3} - 120x_{3n+3} + 3(22y_{n+1} - 120x_{n+1})$ is a cubic integer.
4. $S_f = 6[22(y_{2n+2} - y_{n+1}) - 120(x_{2n+2} - x_{n+1})] + 13$.
5. $GNO_f = 44y_{n+1} - 240x_{n+1} - 1$.
6. $6P_f^m = 22[(m-2)y_{3n+3} + 3y_{2n+2} + (2m-1)y_{n+1}] - 120[(m-2)x_{3n+3} + 3x_{2n+2} + (2m-1)x_{n+1}] + 6, m\ \ 3$.
7. $6CP_f^m = 22[m(y_{3n+3} + 2y_{n+1}) + 6y_{n+1}] - 120[m(x_{3n+3} + 2x_{n+1}) + 6x_{n+1}]$.
8. $2t_{m,f} = 22[(m-2)y_{2n+2} + (4-m)y_{n+1}] - 120[(m-2)x_{2n+2} + (4-m)\ x_{n+1}] + 2m - 4, m\ \ 3$.

9. $$2CP_{m,f} = m[(m-2)(22y_{2n+2} - 120x_{2n+2}) + (4-m)(22y_{n+1} - 120x_{n+1}) + 2(m-2)] + 2m3$$

10. Let $Y = 22y_{n+1} - 120x_{n+1}$ and $X = 2y_{n+1} - 11x_{n+1}$. Then the pair (X,Y) satisfies the hyperbola $Y^2 = 120X^2 + 4$.

**REMARKABLE OBSERVATIONS**

1. Let α be any non-zero positive integer such that $\alpha_s = \frac{y_s - 1}{2}, s = 0,1,2, \ldots$, it is seen that $40t_{3,\alpha_s}$ is a Nasty Number.
2. Let p, q be the generators of the pythogorean triangle $T(\propto, \beta, \gamma)$ with $\propto = 2pq, \beta = p^2 - q^2, \gamma = p^2 + q^2, p > q > 0$. Let $q_s = x_s, p_s = x_s + y_s$. Then T satisfies the following relations.
   (i) $\propto -15\beta + 14\gamma + 1 = 0.$
   (ii) $\frac{4A}{P} + 1 = 16\beta - 15\gamma$ where A and P represents the area and perimeter of the pythogorean triangle.
   (iii) $16 \propto -\gamma = \frac{60A}{P} - 1.$

**CONCLUSION**

To conclude, one may search for other choices of hyperbolas for patterns of solutions and their corresponding properties.

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