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RESEARCH ARTICLE

OBSERVATIONS ON $Z^2 = 3X^2 + Y^2$

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ABSTRACT

The ternary quadratic equation given by $Z^2 = 3X^2 + Y^2$ is considered. Employing its non-zero integral solutions, relations among few special polygonal numbers are determined.

INTRODUCTION

In [1-3], different patterns of m-gonal numbers are presented. In [4] explicit formulas for the rank of Triangular numbers which are simultaneously equal to Pentagonal, Decagonal and Dodecagonal numbers in turn are presented. In [5] the relations among the pairs of special m-gonal numbers generated through the solutions of the binary quadratic equations are determined. In [6] the relations among few polygonal and centered polygonal numbers are determined.

In this communication, we consider the ternary quadratic equation given by $Z^2 = 3X^2 + Y^2$ and obtain the relations among the pairs of special m-gonal numbers generated through its solutions.

KEYWORDS&PHRASES: Pell equations, Ternary quadratic equation.

2010 Mathematics subject classification: **11D09**

NOTATIONS: $T_{m,n}$: Polygonal number of rank n with m sides

METHOD OF ANALYSIS:

Consider the Diophantine equation

$$Z^2 = 3X^2 + Y^2 \tag{1}$$

whose general solutions are

$$\left.\begin{aligned} X &= 2pq \\ Y &= 3p^2 - q^2 \\ Z &= 3p^2 + q^2 \end{aligned}\right\} \tag{2}$$

where p and q are non-zero positive integers.

CASE (1):

The choice,

$$4M - 1 = 3p^2 + q^2 \text{ , } N - 1 = 3p^2 - q^2 \tag{3}$$

in (1) leads to the relation that

"$8T_{6,M} - T_{4,N} + 1 =$ **3 times a square integer"**

From (3), the values of ranks of the Hexagonal numbers and Square numbers are respectively given by

$$M = \frac{3p^2 + q^2 + 1}{4},\ N = 3p^2 - q^2$$

For integer values of M and N, choose $p = 2k - 1, q = 2k$

EXAMPLES: TABLE: 1

k	M	N	$8T_{6,M} - T_{4,N} + 1$
2	11	11	$3(24)^2$
3	28	39	$3(60)^2$
4	53	83	$3(112)^2$
5	86	143	$3(180)^2$
6	127	219	$3(264)^2$

CASE (2):

The choice,

$$4M - 1 = 3p^2 + q^2 \text{ , } 2N + 1 = 3p^2 - q^2 \tag{4}$$

in (1) leads to the relation that

"$8T_{6,M} - 8T_{3,N} =$ **3 times a square integer"**

From (4), the values of ranks of the Hexagonal numbers and Triangular numbers are respectively given by

$$M = \frac{3p^2 + q^2 + 1}{4},\quad N = \frac{3p^2 - q^2 - 1}{2},$$

For integer values of M and N, choose $p = 2k - 1, q = 2k$

EXAMPLES: TABLE: 2

k	M	N	$T_{3,M}-3T_{5,N}$
2	11	5	$3(24)^2$
3	28	19	$3(60)^2$
4	53	41	$3(112)^2$
5	86	71	$3(180)^2$
6	127	109	$3(264)^2$

CASE (3):

The choice,

$$2M+1=3p^2+q^2\ ,\ N=3p^2-q^2 \qquad (5)$$

in (1) leads to the relation that

"$8T_{3,M}-T_{4,N}+1=$ 3 times a square integer"

From (5), the values of ranks of the Triangular numbers and Square numbers are respectively given by

$$M=\frac{3p^2+q^2-1}{2},\ N=3p^2-q^2$$

For integer values of M and N, choose $p=2k-1$, $q=2k-1$

EXAMPLES: TABLE: 3

k	M	N	$T_{3,M}-T_{4,N}+1$
2	21	11	$3(24)^2$
3	55	39	$3(60)^2$
4	105	83	$3(112)^2$
5	171	143	$3(180)^2$
6	253	219	$3(264)^2$

CASE (4):

The choice,

$$5M-2=3p^2+q^2\ ,\ N=3p^2-q^2 \qquad (6)$$

in (1) leads to the relation that

"$5T_{12,M}-T_{4,N}+4=$ 3 times a square integer"

From (6), the values of ranks of the numbers and Square numbers are respectively given by

$$M=\frac{3p^2+q^2+2}{5},\ N=3p^2-q^2$$

For integer values of M and N, choose $p=5k+1, q=5k$

EXAMPLES: TABLE: 4

k	M	N	$5T_{12,M}-T_{4,N}+4$
1	27	83	$3(60)^2$
2	93	263	$3(220)^2$
3	199	543	$3(480)^2$
4	345	923	$3(840)^2$
5	531	1403	$3(1300)^2$

CASE (5):

The choice,

$$4M-1=3p^2+q^2\ ,\ 3N-1=3p^2-q^2 \quad (7)$$

in (1) leads to the relation that

" $8T_{6,M}-3T_{8,N}$ = 3 times a square integer "

From (7), the values of ranks of the Hexagonal numbers and octagonal numbers are respectively given by

$$M=\frac{3p^2+q^2+1}{4},\ N=\frac{3p^2-q^2+1}{3}$$

For integer values of M and N, choose $p=6k-3, q=6k-4$

EXAMPLES: TABLE: 5

k	M	N	$8T_{6,M}-3T_{8,N}$
1	8	8	$3(12)^2$
2	77	60	$3(144)^2$
3	218	160	$3(420)^2$
4	431	308	$3(840)^2$
5	716	504	$3(1404)^2$

CASE (6):

The choice,

$$2M+1=3p^2+q^2\ ,\ 3N-1=3p^2-q^2 \quad (8)$$

in (1) leads to the relation that

" $8T_{3,M}-3T_{8,N}$ = 3 times a square integer "

From (8), the values of ranks of the Triangular numbers and octangular numbers are respectively given by

$$M=\frac{3p^2+q^2-1}{2},\ N=\frac{3p^2-q^2+1}{3}$$

For integer values of M and N, choose $p=3k, q=3k-1$

EXAMPLES: TABLE: 6

k	M	N	$8T_{3,M}-3T_{8,N}$
1	15	8	$3(12)^2$
2	66	28	$3(60)^2$
3	153	60	$3(144)^2$
4	276	104	$3(264)^2$
5	435	160	$3(420)^2$

CASE (7):

The choice,

$$M=3p^2+q^2,\ 3N-1=3p^2-q^2 \tag{9}$$

in (1) leads to the relation that

"$T_{4,M}-3T_{8,N}-1=$ **3 times a square integer** "

From (9), the values of ranks of the Square numbers and octagonal numbers are respectively given by

$$M=3p^2+q^2,\ N=\frac{3p^2-q^2+1}{3}$$

For integer values of M and N, choose $p=3k, q=3k-1$

EXAMPLES: TABLE: 7

k	M	N	$T_{4,M}-3T_{8,N}-1$
2	133	28	$3(60)^2$
3	307	60	$3(144)^2$
4	553	104	$3(264)^2$
5	871	160	$3(420)^2$
6	1261	228	$3(612)^2$

CASE (8):

The choice,

$$5M-2=3p^2+q^2,\ 2N+1=3p^2-q^2 \tag{10}$$

in (1) leads to the relation that

"$5T_{12,M} - 8T_{3,N} + 3 =$ 3 times a square integer "

From (10), the values of ranks of the dodecagonal numbers and Triangular numbers are respectively given by

$$M = \frac{3p^2 + q^2 + 2}{5}, \quad N = \frac{3p^2 - q^2 - 1}{2}$$

For integer values of M and N, choose $p = 5k - 2, q = 5k - 1$

EXAMPLES: TABLE: 8

k	M	N	$5T_{12,M} - 8T_{3,N} + 3$
1	9	5	$3(24)^2$
2	55	55	$3(144)^2$
3	141	155	$3(364)^2$
4	267	305	$3(684)^2$
5	433	505	$3(1104)^2$

CONCLUSION

To conclude, we may search for other relations to (1) by using special polygonal numbers.

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