International Journal of Engineering Research-Online
A Peer Reviewed International Journal
Articles available online http://www.ijoer.in

Vol.2., Issue.1., 2014

RESEARCH ARTICLE

ISSN:2321-7758

NON-HOMOGENEOUS QUINTIC EQUATION WITH FIVE UNKNOWNS $\mathbf{X^4 - Y^4 = 10(Z^2 - W^2)T^3}$

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Article Received: 28/02/2014 | Article Revised on: 10/03/2014 | Article Accepted on:11/03/2014

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ABSTRACT

We obtain infinitely many non-zero integer quintuples (X,Y,Z,W,T) satisfying the Quintic equation with five unknowns $X^4 - Y^4 = 10(Z^2 - W^2)T^3$. Various interesting properties between the values of X, Y, Z, W, T and special Polygonal and Pyramidal numbers are presented..

INTRODUCTION

The theory of Diophantine equation offers a rich variety of fascinating problems. In particular, Quintic equations, homogeneous and non-homogeneous have aroused the interest of numerous mathematicians since antiquity [1-3]. For illustration, one may refer [4-10] for Quintic equation with five unknowns. This paper concerns with the problem of determining non-trivial integral solutions of the non-homogeneous Quintic equation with five unknowns given by $X^4 - Y^4 = 10(Z^2 - W^2)T^3$. A few relations among the solutions are presented.

Keywords: Quintic equation with five unknowns, integer solutions.

2010 Mathematics Subject Classification: 11D41

Notations:

- $T_{m,n}$: Polygonal number of rank n with m sides
- P_n^m : Pyramidal number of rank n with m sides
- Pr_n : Pronic number of rank n
- OH_n : Octahedral number of rank n
- SO_n : Stella Octangular number of rank n
- $Ct_{4,n}$: Centered square number of rank n
- $CP_{6,n}$: Centered hexagonal Pyramidal number of rank n
- $CP_{8,n}$: Centered octagonal Pyramidal number of rank n
- $F_{6,n,3}$: Six dimensional Figurate number of rank n whose generating polygon is a triangle
- $F_{5,n,3}$: Five dimensional Figurate number of rank n whose generating polygon is a triangle
- $F_{4,n,3}$: Four dimensional Figurate number of rank n whose generating polygon is a triangle
- $F_{3,n,3}$: Three dimensional Figurate number of rank n whose generating polygon is a triangle
- $F_{6,n,4}$: Six dimensional Figurate number of rank n whose generating polygon is a square
- $F_{5,n,4}$: Five dimensional Figurate number of rank n whose generating polygon is a square
- $F_{4,n,4}$: Four dimensional Figurate number of rank n whose generating polygon is a square
- $F_{3,n,4}$: Three dimensional Figurate number of rank n whose generating polygon is a square

METHOD OF ANALYSIS:

The Quintic equation with five unknowns to be solved is

$$X^4 - Y^4 = 10(Z^2 - W^2)T^3 \tag{1}$$

The processes of obtaining patterns of integral solutions to (1) are illustrated below.

Introducing the transformations $X = u + v, Y = u - v, Z = 2uv + 1, W = 2uv - 1, u \neq v$ (2)

in (1), it is written as

$$u^2 + v^2 = 10T^3 \tag{3}$$

Assume $T(a,b) = a^2 + b^2, a,b \neq 0$ (4)

Here, we present five different choices of solutions of (3) and hence, obtain five different patterns of solutions to (1).

PATTERN:1

Write 10 as

$$10 = (3+i)(3-i) \tag{5}$$

Using (4) and (5) in (3) and employing the method of factorization, define

$$u + iv = (3+i)(a+ib)^2 \tag{6}$$

Equating the real and imaginary parts on both sides, we get

$$u = 3a^3 - 9ab^2 - 3a^2b + b^3$$

$$v = a^3 - 3ab^2 + 9a^2b - 3b^3$$

Substituting the values of u and v in (2), we get

$$\left.\begin{aligned}
X &= X(a,b) = 4a^3 - 2b^3 - 12ab^2 + 6a^2b \\
Y &= Y(a,b) = 2a^3 + 4b^3 - 6ab^2 - 12a^2b \\
Z &= Z(a,b) = 6(a^6 - b^6) + 48(ab^5 - a^5b) + 90(a^2b^4 - a^4b^2) - 160a^3b^3 + 1 \\
W &= W(a,b) = 6(a^6 - b^6) + 48(ab^5 - a^5b) + 90(a^2b^4 - a^4b^2) - 160a^3b^3 - 1
\end{aligned}\right\} \quad (7)$$

Thus (4) and (7) represent the non-zero distinct integer solutions of (1).

Properties:

- $X(a,1) - 8P_a^5 - T_{6,a} \equiv 2 \pmod{11}$
- $24P_b^3 - Y(1,b) - 36T_{3,b} \equiv 0 \pmod{2}$
- $W(a,1) - 4320F_{6,a,3} + 16560F_{5,a,3} - 18720F_{4,a,3} + 680SO_a + 3224\Pr_a \equiv 3 \pmod{4}$
- $X(a,1) - 24F_{3,a,3} + T_{14,a} \equiv 1 \pmod{2}$

PATTERN:2

Instead of (5), write 10 as

$$10 = (1+3i)(1-3i) \quad (8)$$

Following the procedure presented in pattern:1, the corresponding values of X,Y,Z and W satisfying (1) are

$$\left.\begin{aligned}
X &= X(a,b) = 4a^3 + 2b^3 - 12ab^2 - 6a^2b \\
Y &= Y(a,b) = -2a^3 + 4b^3 + 6ab^2 - 12a^2b \\
Z &= Z(a,b) = 6(a^6 - b^6) - 48(ab^5 + a^5b) + 90(a^2b^4 - a^4b^2) + 160a^3b^3 + 1 \\
W &= W(a,b) = 6(a^6 - b^6) - 48(ab^5 + a^5b) + 90(a^2b^4 - a^4b^2) + 160a^3b^3 - 1
\end{aligned}\right\} \quad (9)$$

Thus (4) and (9) represent the non-zero distinct integer solutions of (1).

Properties:

- $6(OH_a) - X(a,1) - T_{14,a} \equiv -2 \pmod{19}$
- $Y(1,b) - SO_b - T_{14,b} \equiv 2 \pmod{5}$
- $Z(1,b) + 2160F_{6,b,4} - 1440F_{5,b,4} - 228F_{4,b,3} + 3120F_{3,b,4} - T_{1190,b} \equiv 1 \pmod{3}$
- $X(a,1) - 12F_{3,a,4} + T_{38,a} \equiv -1 \pmod{2}$

PATTERN:3

(3) Can be written as $u^2 + v^2 = 10T^3 * 1$ (10)

Write 1 and 10 as

$$\left.\begin{aligned}
1 &= \frac{(4+3i)(4-3i)}{25} \\
10 &= (3+i)(3-i)
\end{aligned}\right\} \quad (11)$$

Substituting (4) and (11) in (10) and employing the method of factorization, define

$$u + iv = \frac{(4+3i)(3+i)}{5}(a+ib)^3$$

Equating the real and imaginary parts on both sides, we get

$$u=\frac{1}{5}[9a^3-27ab^2-39a^2b+13b^3]$$

$$v=\frac{1}{5}[13a^3-39ab^2+27a^2b-9b^3]$$

Substituting the values of u and v in (10), we get

$$\left.\begin{aligned}X&=X(a,b)=\frac{1}{5}[22a^3+4b^3-66ab^2-12a^2b]\\Y&=Y(a,b)=\frac{1}{5}[-4a^3+22b^3+12ab^2-66a^2b]\\Z&=Z(a,b)=\frac{1}{25}[234(a^6-b^6)-528(a^5b+ab^5)-3510(a^4b^2-a^2b^4)+1760a^3b^3]+1\\W&=W(a,b)=\frac{1}{25}[234(a^6-b^6)-528(a^5b+ab^5)-3510(a^4b^2-a^2b^4)+1760a^3b^3]-1\end{aligned}\right\}\quad(12)$$

As our interest is on finding integer solutions, we choose a and b suitably so that the values of X,Y.Z,W and T are integers.

Replacing a by 5A and b by 5B in (4) and (12), the corresponding integer solutions of (1) in two parameters are

$$X=X(A,B)=550A^3+100B^3-1650AB^2-300A^2B$$

$$Y=Y(A,B)=-100A^3+550B^3+300AB^2-1650A^2B$$

$$Z=Z(A,B)=234(625A^6-625B^6)-528(625A^5B+625AB^5)-3510(625A^4B^2-625A^2B^4)+1760(625A^3B^3)+1$$

$$W=W(A,B)=234(625A^6-625B^6)-528(625A^5B+625AB^5)-3510(625A^4B^2-625A^2B^4)+1760(625A^3B^3)-1$$

$$T=T(A,B)=25A^2+25B^2$$

Properties:

- $600P_B^3-X(1,B)-1950Pr_B\equiv 0(\bmod 5)$
- $300P_A^4-Y(-A,1)-1800Pr_A\equiv 0(\bmod 10)$
- $X(A,1)-1100P_A^5+T_{1702,A}\equiv 0(\bmod 2)$
- $Y(1,B)-T(1,B)-3300F_{3,B,3}+2750T_{3,B}\equiv 0(\bmod 5)$

PATTERN:4

Instead of (11), write 1 and 10 as

$$\left.\begin{aligned}10&=(1+3i)(1-3i)\\1&=\frac{(3+4i)(3-4i)}{25}\end{aligned}\right\}\quad(13)$$

Following the procedure presented in pattern:3, the corresponding values of X,Y,Z,W and T satisfying (1) are

$$X=X(A,B)=100A^3+550B^3-300AB^2-1650A^2B$$

$$Y=Y(A,B)=-550A^3+100B^3+1650AB^2-300A^2B$$

$$Z=Z(A,B)=234(625B^6-625A^6)-528(625A^5B+625AB^5)+3510(625A^4B^2-625A^2B^4)+1760(625A^3B^3)+1$$

$$W=W(A,B)=234(625B^6-625A^6)-528(625A^5B+625AB^5)+3510(625A^4B^2-625A^2B^4)+1760(625A^3B^3)-1$$

$$T=T(A,B)=25A^2+25B^2$$

Properties:

- $50SO_A-X(A,1)-1650Pr_A\equiv 0(\bmod 2)$

- $225SO_A - X(A,1) - Y(A,1) - 1950\Pr_A \equiv 0 \pmod 5$
- $X(1,B) - 825(OH_B) + 300\Pr_B \equiv 0 \pmod 5$
- $Y(A,1) + 1500F_{3,A,4} - 600Ct_{4,A} \equiv 0 \pmod 2$

PATTERN:5

In addition to (11) and (13), write 1 and 10 as

$$\left.\begin{aligned} 1 &= \frac{(4+3i)(4-3i)}{25} \\ 10 &= (1+3i)(1-3i) \end{aligned}\right\} \quad (14)$$

Substituting (4) and (11) in (10) and employing the method of factorization, define

$$u + iv = \frac{(1+3i)(4+3i)}{5}(a+ib)^3 \quad (15)$$

Equating the real and imaginary parts on both sides, we get

$$u = -a^3 + 3ab^2 - 9a^2b + 3b^3$$

$$v = 3a^3 - 9ab^2 - 3a^2b + b^3$$

Substituting the values of u and v in (10), we get

$$\left.\begin{aligned} X &= X(a,b) = 2a^3 + 4b^3 - 6ab^2 - 12a^2b \\ Y &= Y(a,b) = -4a^3 + 2b^3 + 12ab^2 - 6a^2b \\ Z &= Z(a,b) = 6(b^6 - a^6) - 48(ab^5 + a^5b) + 90(a^4b^2 - a^2b^4) + 160a^3b^3 + 1 \\ W &= W(a,b) = 6(b^6 - a^6) - 48(ab^5 + a^5b) + 90(a^4b^2 - a^2b^4) + 160a^3b^3 - 1 \end{aligned}\right\} \quad (16)$$

Thus (4) and (16) represents the non-zero distinct integer solutions of (1).

Properties:

- $6P_a^4 - X(a,1) - T_{32,a} \equiv 1 \pmod 4$
- $Y(1,b) - 4P_b^5 - T_{22,b} \equiv -1 \pmod 3$
- $X(1,b) + 6CP_{8,b} - T_{14,b} \equiv -1 \pmod 2$
- $Z(a,1) + 4320F_{6,a,3} - 2520F_{5,a,4} - 3168F_{4,a,4} + 776CP_{6,a} + 1734\Pr_a \equiv 0 \pmod 6$

CONCLUSION

First of all, it is worth to mention here that in (2), the values of Z and W may also be represented by $Z = 2u + v, W = 2u - v$ and thus will obtain other choices of solutions to (1). In conclusion, one may consider other forms of Quintic equation with five unknowns and search for their integer solutions.

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