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RESEARCH ARTICLE

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ON THE BIQUADRATIC EQUATION WITH FOUR UNKNOWNS

$$x^3 + y^3 = 39zw^3$$

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ABSTRACT

We obtain infinitely many non-zero integer quadruples (x, y, z, w) satisfying the the Biquadratic equation with four unknowns $x^3 + y^3 = 39zw^3$. Various interesting properties among the values of x, y, z and w are presented.

Keywords: Biquadratic equation with four unknowns, integral solutions
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INTRODUCTION

The theory of Diophantine equations offers a rich variety of fascinating problems. In particular, biquadratic diophantine equations, homogeneous and non-homogeneous have aroused the interest of numerous mathematicians since antiquity [1-5]. In this context, one may refer [6-11] for various problems on the biquadratic diophantine equations with four variables. However, often we come across non-homogeneous biquadratic equations and as such one may require its integral solution in its most general form. It is towards this end, this paper concerns with the problem of determining non-trivial integral solutions of the non-homogeneous equation with four unknowns given by $x^3 + y^3 = 39zw^3$. A few relations among the solutions are presented.

NOTATIONS:

P_n^m - Pyramidal number of rank n with size m.

$T_{m,n}$ - Polygonal number of rank n with size m.

SO_n - Stella Octangular number of rank n

OH_n - Octahedral number of rank n

2. METHOD OF ANALYSIS:

The biquadratic equation with four unknowns under consideration is

$$x^3 + y^3 = 39zw^3 \tag{1}$$

Different patterns of solutions for (1) are illustrated below:

2.1 PATTERN: 1

Introducing the linear transformations,

$$x = u + v, \quad y = u - v, \quad z = 2u, \quad w = a^2 + 3b^2 \tag{2}$$

in (1), it becomes

$$u^2 + 3v^2 = 39(a^2 + 3b^2)^3 \tag{3}$$

Write 39 as

$$39 = (6 + i\sqrt{3})(6 - i\sqrt{3}) \tag{4}$$

Using (4) in (3) and employing the method of factorization and equating positive factors we get,

$$(u + i\sqrt{3}v) = (6 + i\sqrt{3})(a + i\sqrt{3}b)^3 \tag{5}$$

Equating real and imaginary parts of (5), we get

$$u = u(a,b) = 6a^3 - 9a^2b - 54ab^2 + 9b^3$$
$$v = v(a,b) = a^3 + 18a^2b - 9ab^2 - 18b^3$$

Employing (2), the values of x, y, z and w satisfying (1) are given by

$$x(a,b) = u + v = 7a^3 + 9a^2b - 63ab^2 - 9b^3$$
$$y(a,b) = u - v = 5a^3 - 27a^2b - 45ab^2 + 27b^3$$
$$z(a,b) = 2u = 12a^3 - 18a^2b - 108ab^2 + 18b^3$$
$$w(a,b) = a^2 + 3b^2$$

PROPERTIES:

- $y(a,1) + w(a,1) - 10P_a^5 + 62T_{3,a} \equiv 0 \pmod{2}$
- $x(1,b) - y(1,b) + 18SO_b + 36T_{3,b} \equiv 0 \pmod{2}$
- $42P_a^3 - x(a,1) - T_{26,a} \equiv 9 \pmod{88}$
- $z(1,b) - 9SO_b + T_{218,b} \equiv 0 \pmod{2}$

2.2 PATTERN: 2

One may write (3) as,

$$u^2 + 3v^2 = 39(a^2 + 3b^2)^3 * 1 \tag{6}$$

Write 1 as

$$1=\frac{\left(1+i\sqrt{3}\right)\left(1-i\sqrt{3}\right)}{4} \tag{7}$$

Using (4) and (7) in (6) and employing the method of factorization and equating the positive factors we get,

$$\left(u+i\sqrt{3}v\right)=\frac{1}{2}\left(6+i\sqrt{3}\right)\left(1+i\sqrt{3}\right)\left(a+i\sqrt{3}b\right)^{3} \tag{8}$$

Equating real and imaginary parts of (8), we get

$$u=u(a,b)=\frac{1}{2}\left[3a^{3}-63a^{2}b-27ab^{2}+63b^{3}\right]$$

$$v=v(a,b)=\frac{1}{2}\left[7a^{3}+9a^{2}b-63ab^{2}-9b^{3}\right]$$

Employing (2), the values of x, y, z and w satisfying (1) are given by

$$x(a,b)=u+v=5a^{3}-27a^{2}b-45ab^{2}+27b^{3}$$

$$y(a,b)=u-v=-2a^{3}-36a^{2}b+18ab^{2}+36b^{3}$$

$$z(a,b)=2u=3a^{3}-63a^{2}b-27ab^{2}+63b^{3}$$

$$w(a,b)=a^{2}+3b^{2}$$

PROPERTIES:

- $72P_b^5-y(1,b)-T_{38,b}\equiv 2\pmod{53}$
- $30P_a^3-x(a,1)-84T_{3,a}\equiv -27\pmod{13}$
- $z(1,b)-w(1,b)-126P_b^5+186T_{3,b}\equiv 0\pmod{2}$
- $w(a,1)-y(a,1)-3OH_a-T_{76,a}\equiv -33\pmod{17}$

REMARK:

It is worth to note that 39 in (4) and 1 in (7) are also represented in the following ways:

$$39=\frac{\left(3+i7\sqrt{3}\right)\left(3-i7\sqrt{3}\right)}{4}=\frac{\left(9+i5\sqrt{3}\right)\left(9-i5\sqrt{3}\right)}{4}$$

$$1=\frac{\left(1+i4\sqrt{3}\right)\left(1-i4\sqrt{3}\right)}{49}=\frac{\left(11+i4\sqrt{3}\right)\left(11-i4\sqrt{3}\right)}{169}$$

By introducing the above representations in (4) and (7), one may obtain 4 different patterns of solutions to (1)

3. CONCLUSION:

In (2), the representation for z may be taken as $z=2^{2k+1}u$. To conclude, one may search for other patterns of non-zero integer distinct solutions and their corresponding properties

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