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ON TERNARY QUADRATIC DIOPHANTINE EQUATION $47X^2+Y^2=Z^2$

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ABSTRACT

The ternary quadratic homogeneous equation representing a cone given by $47X^2+Y^2=Z^2$ is analyzed for its non-zero distinct integer points on it. Six different patterns of integer solutions satisfying the cone under consideration are given. A few interesting relations between the solutions and special number patterns are presented. Given an integral solution on the considered cone, three triples of integers generated from the given solution are exhibited.

Keywords: Ternary quadratic, integral solutions

2010 Mathematics Subject Classification: 11D09

Notations:

P_{nm} : Pyramid number of rank n with size m

$T_{m,n}$: Polygonal number of rank n with size m

SO_n: Stella octangular number of rank m

INTRODUCTION

The ternary quadratic Diophantine equations offer an unlimited field for research due to their variety [1, 21]. For an extensive review of various problems, one may refer [2-20]. This communication concerns with yet another interesting ternary quadratic equation $47X^2+Y^2=Z^2$ for determining its infinitely many non-zero integral points. Also, a few interesting relations among the solutions and special numbers are presented

METHOD OF ANALYSIS:

The ternary quadratic equation under consideration is

$$47x^2 + y^2 = z^2 \quad (1)$$

To start with it is seen that the triples

$(k, 23k, 24k), (2k+1, 2k^2+2k-23, 2k^2+2k+24)$ and

$(2rs, r^2-47s^2, r^2+47s^2)$ satisfy (1).However, we have other choices of solutions to (1) which are illustrated below:

Consider (1) as

$$y^2 + 47x^2 = z^2 * 1 \quad (2)$$

Assume $\quad z = a^2 + 47b^2 \quad (3)$

Write 1 as

$$1 = \frac{\left((23+2n-2n^2) + i(2n-1)\sqrt{47}\right)\left((23+2n-2n^2) - i(2n-1)\sqrt{47}\right)}{\left[24-2n+2n^2\right]^2} \quad (4)$$

Substituting (3) and (4) in (2) and employing the method of factorization, define

$$y + i\sqrt{47}\,x = \left(a + i\sqrt{47}\,b\right)^2 \frac{((23+2n-2n^2) + i(2n-1)\sqrt{47})}{(24-2n+2n^2)}$$

Equating the real and imaginary parts in the above equation, we get

$$x = \frac{((a^2-47b^2)(2n-1)) + 2(23+2n-2n^2)ab}{(2n^2-2n+24)}$$

$$y = \frac{((23+2n-2n^2)(a^2-47b^2)) - 94(2n-1)ab}{(2n^2-2n+24)}$$

Replacing a by $(2n^2-2n+24)A$,b by $(2n^2-2n+24)B$ in the above equations the corresponding integer solutions to (1) are given by

$$\left.\begin{aligned} x &= \{(2n-1)(2n^2-2n+24)(A^2-47B^2)\} + \{2*(23+2n-2n^2)(24-2n+2n^2)\}AB \\ y &= \{(2n^2-2n+24)(23+2n-2n^2)(A^2-47B^2)\} - \{94(2n-1)(24-2n+2n^2)\}AB \\ z &= (2n^2-2n+24)^2(A^2+47B^2) \end{aligned}\right\} \quad (A)$$

For simplicity and clear understanding, taking $n=1$ in (A), the corresponding integer solutions of (1) are given by

$$x = 24A^2 + 1104AB - 1128B^2$$
$$y = 552A^2 - 2256AB - 25944B^2$$
$$z = 24^2(A^2+47B^2)$$

Properties:

- $x(A,1) - T_{50,A} \equiv -1 \pmod{1127}$
- $x(A,1) - T_{30,A} - T_{22,A} \equiv 0 \pmod 2$
- $y(A,1) - 23x(A,1) \equiv 0 \pmod{27648}$
- $y(1,A) - x(1,A) - T_{6724,A} + T_{56356,A} \equiv 528 \pmod{28176}$
- $24x(A,A+1) + 47z(A,A+1) - 92T_{3,A} - T_{4422,A} \equiv 0 \pmod{2209}$

- $24x(A,2A^2-1)-46So_A+188T_{4,A^2}-T_{380,A}\equiv -47(\bmod 188)$
- $24x(A,2A^2+1)-138OH_A+188T_{4,A^2}+T_{374,A}\equiv -47(\bmod 186)$

It is worth to note that 1 in (2) may also be represented as

$$1=\frac{((47-4n^2)+i4n\sqrt{47})((47-4n^2)-i4n\sqrt{47})}{(47+4n^2)^2}$$

Following the analysis presented above, the corresponding integer solutions to (1) are found to be

$$\left.\begin{aligned} x&=\{4n(47+4n^2)(A^2-47B^2)\}+\{2(47-4n^2)(47+4n^2)AB\}\\ y&=\{(4n^2+47)(47-4n^2)(A^2-47B^2)\}-\{376n(47+4n^2)AB\}\\ z&=(47+4n^2)^2(A^2+47B^2)\end{aligned}\right\} \quad \text{(B)}$$

For the sake of simplicity, taking n=1 in (B), the corresponding integer solutions of (1) are given by

$$x=204A^2-9588B^2+4386AB$$
$$y=2193A^2-19176AB-103071B^2$$
$$z=51^2(A^2+47B^2)$$

Properties:

- $x(A,1)-T_{410,A}\equiv -1222(\bmod 4138)$
- $2Z(A,A)=6(4A^2)$ Is a nasty number
- $z(A,1)-T_{96,A}\equiv 1(\bmod 46)$
- $y(A,1)-x(A,1)-9T_{444,A}\equiv -239(\bmod 2398)$
- $y(A,1)-3T_{1464,A}\equiv 1155(\bmod 5662)$
- $x(A,1)-T_{202,A}-T_{210,A}\equiv -312(\bmod 4588)$
- $51y(n,n+1)+376pr_n-T_{3958,n}\equiv -2021(\bmod 6019)$
- $51y(A^2,A+1)-752p_A^5-43T_{4,A^2}+T_{4044,A}\equiv -2021(\bmod 6063)$

GENERATION OF INTEGER SOLUTIONS

Let (x_0,y_0,z_0) be any given integer solution of (1)

Then, each of the following triples of integers satisfies (1):

Triple 1: $(95x_0+2y_0-14z_0, 94x_0+3y_0-14z_0, 658x_0+14y_0-97z_0)$

Triple 2: (x_{n2},y_{n2},z_{n2})

$$x_{n2}=\frac{1}{46}\left[\{47(-23)^n-(23)^n\}x_0+\{(23)^n-(-23)^n\}z_0\right]$$

$$y_{n2}=23^n y_0$$

$$z_{n2}=\frac{1}{46}\left[\{47(-23)^n-47(23)^n\}x_0+\{47(23)^n-(-23)^n\}z_0\right]$$

Triple 3: (x_{n3}, y_{n3}, z_{n3})

$$x_{n3} = \frac{1}{48}\left[\{(24)^n + 47(-24)^n\}x_0 + \{(-24)^n - (24)^n\}y_0\right]$$

$$y_{n3} = \frac{1}{48}\left[\{47(-24)^n - 47(24)^n\}x_0 + \{47(24)^n + (-24)^n\}y_0\right]$$

$$z_{n3} = 24^n z_0$$

Triple 4: (x_{n4}, y_{n4}, z_{n4})

$$x_{n4} = 3^n x_0$$

$$y_{n4} = \frac{1}{6}\left[\{8(3)^n - 2(-3)^n\}y_0 + \{4(-3)^n - 4(3)^n\}z_0\right]$$

$$z_{n4} = \frac{1}{6}\left[\{4(3)^n - 4(-3)^n\}y_0 + \{8(-3)^n - 2(3)^n\}z_0\right]$$

CONCLUSION

In this dissertation, the ternary quadratic Diophantine equations referring a cone are analyzed for is non-zero distinct integral Points. A few interesting properties between the solutions and special numbers are presented. To conclude, one may search for other patterns of solutions and their corresponding properties for the cone under consideration

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