International journal of Engineering Research-Online
A Peer Reviewed International Journal
Articles available onlne http://www.ijoer.in
Vol.1., Issue.2., 2013

RESEARCH ARTICLE

ISSN: 2321-7758

ON THE QUINTIC EQUATION WITH FIVE UNKNOWNS

$$2(x-y)(x^3+y^3)=19(z^2-w^2)p^3$$

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Article Received: 30/08/2013 | Article Revised on: 12/09/2013 | Article Accepted on:04/09/2013

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ABSTRACT: We obtain infinitely many non-zero integer (x, y, z, w, p) satisfying the quintic equation $2(x-y)(x^3+y^3)=19(z^2-w^2)p^3$. Various interesting properties among the values of x, y, z, w and p are presented.

Notations:

$t_{m,n}$ - Polygonal number of rank n with size m

$CP_{n,m}$ - Centred polygonal number of rank n with size m

CP_n^m - Central Pyramidal number of rank n with size m.

J_n -Jacobsthal number of rank n

j_n - Jacobsthal-Lucas number of rank n

pr_n - Pronic number of rank n

INTRODUCTION

The theory of Diophantine equations offers a rich variety of fascinating problems. In particular, quintic equations homogeneous or non-homogeneous have aroused the interest of numerous mathematicians since antiquity [1-3]. For illustration, one may refer [4-8] for quintic equation with five unknowns. This paper concerns with the problem of determining non-trivial integral solutions of the non-homogeneous quintic equation with five unknowns given by $2(x-y)(x^3+y^3)=19(z^2-w^2)p^3$. A few relations between the solutions and special numbers are presented.

Method of analysis:

The quintic equation with five unknowns to be solved is $2(x-y)(x^3+y^3)=19(z^2-w^2)p^3$ (1).

The processes of obtaining patterns of integral solutions to (1) are illustrated below:

Pattern:1

Introduction of the transformations

$$x = u + v, \quad y = u - v, \quad z = 2u + v, \quad w = 2u - v, \tag{2}$$

in (1) leads to

$$u^2 + 3v^2 = 19p^3 \tag{3}$$

Let $p = a^2 + 3b^2$ (4)

Write 19 as $19 = \frac{(1+i5\sqrt{3})(1-i5\sqrt{3})}{4}$ (5)

Using (4) and (5) in (3) and applying the method of factorization, define

$$u + i\sqrt{3}v = \frac{(1+i5\sqrt{3})}{2}(a + ib\sqrt{3})^3 \tag{6}$$

Equating real and imaginary parts, we have

$$\left.\begin{aligned} u &= \frac{1}{2}[(a^3 - 9ab^2) - 45(a^2b - b^3)] \\ v &= \frac{1}{2}[5(a^3 - 9ab^2) + 3(a^2b - b^3)] \end{aligned}\right\} \tag{7}$$

Thus, replacing a by 2A and b by 2B in (7) and using (2), the corresponding non-zero distinct integral solutions to (1) are given by

$$x(A,B) = 4[6(A^3 - 9AB^2) - 42(A^2B - B^3)]$$

$$y(A,B) = 4[-4(A^3 - 9AB^2) - 48(A^2B - B^3)] \; z(A,B) = 4[7(A^3 - 9AB^2) - 87(A^2B - B^3)]$$

$$w(A,B) = 4[-3(A^3 - 9AB^2) - 98(A^2B - B^3)]$$

$$p(A,B) = 4A^2 - 12B^2$$

Properties:

$$(i) y(n,n-1) + w(n,n-1) + z(n,n-1) = -932[t_{6,n} - 6t_{3,n} + 3t_{4,n} + 2]$$

$$(ii) x(n,n+1) + y(n,n+1) - w(n,n+1) = -20[6CP_n^5 + 3CP_n^4 + 2CP_n^3 + 2(S_n + 12\Pr_n - 7t_{4,n})]$$

$$(iii) x(n-1,n+1) + y(n-1,n+1) + z(n-1,n+1) - w(n-1,n+1)$$

$$= 4\{12(-3CP_n^6 - t_{16,n} + 2t_{3,n} - 6t_{4,n} + 8) + 316\Pr_n\}$$

Pattern:2

(3) can be written as $u^2 + 3v^2 = 19p^3 * 1$ (8)

Write 1 as $1 = \frac{(1+i\sqrt{3})(1-i\sqrt{3})}{4}$ (9)

Substituting (4), (5) and (9) in (8) and employing the method of factorization, define

$$u + i\sqrt{3}v = \frac{(1+i5\sqrt{3})}{2}(a + ib\sqrt{3})^3 \frac{(1+i\sqrt{3})}{2} \tag{10}$$

Proceeding as in pattern.1, the corresponding non-zero distinct integral solutions to (1) are given by

$$x(A,B) = 2[-8(A^3 - 9AB^2) - 96(A^2B - B^3)]$$
$$y(A,B) = 2[-20(A^3 - 9AB^2) - 12(A^2B - B^3)]$$
$$z(A,B) = 2[-22(A^3 - 9AB^2) - 150(A^2B - B^3)]$$
$$w(A,B) = 2[-34(A^3 - 9AB^2) - 66(A^2B - B^3)]$$
$$p(A,B) = 4A^2 - 12B^2$$

Properties:

$$(i) x(2^n, 2^{n-1}) - z(2^n, 2^{n-1}) = 28 j_{3n} - 426 J_{3n} + 54 J_{2n} - 170(-1)^{3n} - 54$$

$$(ii) x(n, n-10w(n, n-1) = 2[26(-3CP_n^{16} + 2t_{19,n} + Pr_n) - 30(t_{6,n} - 2Pr_n + 2t_{4,n} + 1)]$$

$$(iii) z(n, n-1) - w(n, n-1) = 2[12(8CP_n^6 + 3S_n + 18t_{3,n} - 9t_{4,n} - 3) - 84(CP_{4,n} - 5Pr_n + 5t_{4,n})]$$

Pattern.3

Instead of (9), we write 1 as $1 = \frac{(1 + i4\sqrt{3})(1 - i4\sqrt{3})}{49}$

For this choice the corresponding n on-zero distinct integral solutions to (1) are found to be

$$x(A,B) = 14^2[-50(A^3 - 9AB^2) - 258(A^2B - B^3)]$$
$$y(A,B) = 14^2[-68(A^3 - 9AB^2) + 96(A^2B - B^3)]$$
$$z(A,B) = 14^2[-109(A^3 - 9AB^2) - 339(A^2B - B^3)]$$
$$w(A,B) = 14^2[-127(A^3 - 9AB^2) + 15(A^2B - B^3)]$$
$$p(A,B) = 14^2[A^2 + 3B^2)]$$

Properties:

$$(i) x(n, n+1) + w(n, n+1) = 14^2[177(6CP_n^5 + 6CP_n^8 + CP_{18,n} + 8t_{4,n}) + 243(CP_{4,n} + Pr_n - t_{4,n})]$$

$$(ii) y(n+1, n) - w(n+1, n) = 14^2[59(-3CP_n^{10} - 6CP_n^8 - 7t_{4,n} - 2t_{3,n} + 1) + 81(Pr_n + t_{4,n})]$$

$$(iii) x(n-1, n+1) - z(n-1, n+1) = 14^2[59(-6CP_n^7 - CP_n^{12} - t_{18,n} - 4t_{3,n} + 2t_{4,n} + 10) - 324 Pr_n]$$

CONCLUSION

To start with, it is worth to mention here that, the values of z and w in (2) may be considered as

$$(i) z = u + 2v, \quad w = u - 2v,$$
$$(ii) z = 2uv + 1, \qquad w = 2uv - 1$$

Following the analysis presented above, one may obtain other patterns of non-zero integer solutions to (1)

Further, instead of (5), we write 19 as $19 = (4 + i\sqrt{3})(4 - i\sqrt{3})$. Repeating the process as in pattern.1 one may get yet another different families of non-zero distinct integral solutions to (1)

To conclude, one may search for other choices of transformations to analyze (1) for its non-zero distinct integer solutions.

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