International journal of Engineering Research-Online
A Peer Reviewed International Journal
Articles available onlne http://www.ijoer.in

Vol.1., Issue.2., 2013

RESEARCH ARTICLE

ISSN: 2321-7758

ON THE EXPONENTIAL DIOPHANTINE EQUATION $k^{2z} - xk^{z} = k^{y}$

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Article Received: 20/08/2013 | Revised on: 22/08/2013 | Accepted on: 25/08/2013

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ABSTRACT

Four different infinite families of non-zero distinct integral solutions to the exponential Diophantine equation $k^{2z} - xk^{z} = k^{y}$ are obtained. A few interesting relations between the solutions and special numbers namely, centered polygonal numbers, centered pyramidal numbers, jacobsthal numbers, lucas numbers and kynea numbers are presented.

Keyword: Exponential Diophantine equation, Integral solutions, polygonal numbers and centered polygonal numbers.

MATHEMATICS SUBJECT CLASSIFICATION NUMBER: **11D61**

INTRODUCTION

The exponential Diophantine equation $a^{x} + b^{y} = c^{z}$ where x, y and z are non-negative integers, has been analyzed for various choices of a, b and c by Banyat Sroysang [1-6] and A. Suvarnamani et al [7].. It is well-known that the algebraic equations and in particular, the exponential Diophantine equation are rich in variety. In this paper, a special type of exponential Diophantine equation given by $k^{2z} - xk^{z} = k^{y}$ is analyzed for its positive integral solutions. To be more specific four different infinite families of integer solutions are exhibited.

Notations

$$t_{m,n} = n\left(1 + \frac{(n-1)(m-2)}{2}\right)$$

$$P_n^m = \left(\frac{n(n+1)}{6}\right)[(m-2)n+(5-m)]$$

$$Pt_n = \frac{n(n+1)(n+2)(n+3)}{24}$$

$$SO_n = n(2n^2-1)$$

$$S_n = 6n(n-1)+1$$

$$\Pr_n = n(n+1)$$

$$J_n = \frac{1}{3}\left(2^n-(-1)^n\right)$$

$$j_n = (2^n+(-1)^n)$$

$$Ky_n = (2^n+1)^n - 2.$$

$$F_{4,m,3} = \frac{n(n+1)(n+2)(n+3)}{4!}$$

$$F_{5,m,3} = \frac{n(n+1)(n+2)(n+3)(n+4)}{5!}$$

$$CP_n^m = \frac{mn(n-1)}{2}+1$$

METHOD OF ANALYSIS:

The exponential equation to be solved in integers for x, y, z is

$$k^{2z} - xk^z = k^y \quad (1)$$

To start with it is observed that (1) is satisfied by

$$(x,y,z) = (1-k^\alpha,\alpha,0),(k^\alpha-1,\alpha,\alpha),(0,2\alpha,\alpha)$$

However, we have other choices of non-zero distinct integral solutions to (1) which are illustrated as follows.

Treating (1) as a quadratic in k^z and solving for it, we have,

$$k^z = \frac{1}{2}[x+\sqrt{x^2+4k^y} \quad (2)$$

The process of obtaining the values of x ,y and z is as follows:

CASE:I Let $y = 2\alpha$

Employing the most citied solution of the Pythagorean equation, the following two sets of non-zero distinct integer solution for (1) are obtained.

Set 1: $x = (k^2-1)k^{\alpha-1}$

$z = \alpha+1$

Observations:

I. $2z - y = 2$

II. $k^2x = (k^2-1)k^z$

III. $x = k^z - k^{z-2}$

IV. $x^2k^2 = (k^2-1)^2k^y$

V. $y^2z = 8P_\alpha^5$

VI. $yz = 4t_{3,\alpha}$

VII. $y[z^2-2z+2] = 4CP_{3,\alpha}$

VIII. $6[y^2+4z-3]$ is a nasty number.

Set II: $x = -(k^2-1)k^{\alpha-1}$

$z = \alpha-1$

Observations:

i) $x+(k^2-1)k^z = 0$

ii) $x+k^{y-z}-k^{y-z-2} = 0$

CASE II : Let $y = 2\alpha-1$ .For this choice, two sets of values for x and z are obtained as given below:

Set III : $x = (k-1)k^{\alpha-1}$

$z = \alpha-1$

Observations:

i. $x - k^{z+1} - k^z = 0$

ii. $y^2 + 2z^2 - S_\alpha = 1$

iii. $y^2 z + 3(z+2) - 6OH_\alpha + 2t_{10,\alpha} = 0$

iv. $6[y^2 + 2z + 1]$ is a nasty number.

v. $(y+1)[z^2 + 2z + 2] = 2CP_{3,\alpha}$

Set IV: $x = -(k-1)k^{\alpha-1}$

$z = \alpha - 1$

Observations:

i. $x + k^{z+1} - k^z = 0$

ii. $x + k^{y-z} - k^{y-z-1} = 0$

iii. $(y+1)^2 (z+2) = 8P_\alpha^5$

iv. $y^2 z - 2SO_\alpha + 16t_{3,\alpha} - 15z \equiv 0 (\mathrm{mod}\, 2)$

v. v $y^2 + 2z - 2t_{6,\alpha} + 1 = 0$

CONCLUSION

In this paper the exponential Diophantine equation $k^{2z} - xk^z = k^y$ where k is any non-zero positive integer greater than one has been considered and four different infinite families of non-zero distinct integral solutions are obtained. A few results connecting the integral solutions with special numbers are presented.

One may search for other patterns of solutions along with their properties.

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